

ELEC 3004/7312: Signals, Systems & Controls

Week 4: Tutorial 2: Linear Systems

This tutorial reviews linear systems and some basic operations on them in MATLAB.

Part 1. Getting Started Playing with Matlab's Controls Toolbox

One of MATLAB's distinguishing features is its Controls Toolbox. Let's explore its ability to plot impulse and step responses.

- a) Use `doc control/impulse` and open the documentation page for the step analysis function
- b) Try example 1. Does the plot make sense?
- c) Another way to get the system is to use the `tf` function from the coefficients of the of the Laplacian of the transfer function (as `sys=tf(numerator, denominator)`).
- d) Open `doc control/tf` and use example 1 to build a model.
- e) Now computer the impulse of this model.

Part 2. 2nd and 4th Order Model Response (in MATLAB)

Using the notation above, a second order system maybe written as:

`tf([1],[1 (b/m or mass normalized damping ratio) (k/m or mass normalized stiffness)])`

thus, `impulse (tf(1,[1 0.1 10]))` gives the response of a spring with a little damping.

Let's play with this. In MATLAB try...

```
M = 1;
K = 10;
B = 0.1;
num = 1;
den = [M B K];
sys = tf(num,den);
step(sys);
figure; impulse(sys);
```

Now change the M and K and see the response. Or change the magnitude of the step

```
figure; BIG=100; step(BIG*sys);
```

Note for future reference that discrete time systems can be specified via the `tf` function. The command `tf(num,den,ts)` creates a discrete-time transfer function with sample time T_s (in seconds).

Part 3. Quarter-Car Model (in MATLAB)

From the "Application of H-Infinity and Mu to Active Suspension Control" note (search for car in the MATLAB documentation system, select Quarter Car Model).

See attached PDF from MATLAB Help

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Part 4. Harmonic Oscillator (LC Circuits)

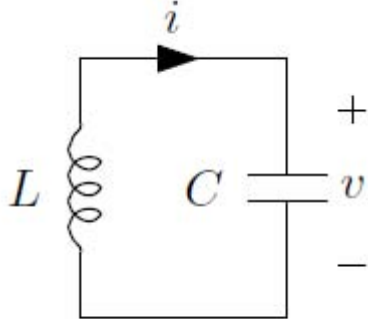
a) Imagine a system described by: $\ddot{y} + \omega^2 y = 0$

The characteristic (Laplace) polynomial is $s^2 + \omega^2$

What are its roots?

What is the solution for $y(t)$?

b) Imagine an LC circuit like:



Verify that $i = C\dot{v}$ and $v = -Li$

c) What is the characteristic equation for the circuit in (b)?

d) Verify that the oscillation frequency for the circuit in (b) is $\omega = \frac{1}{\sqrt{LC}}$

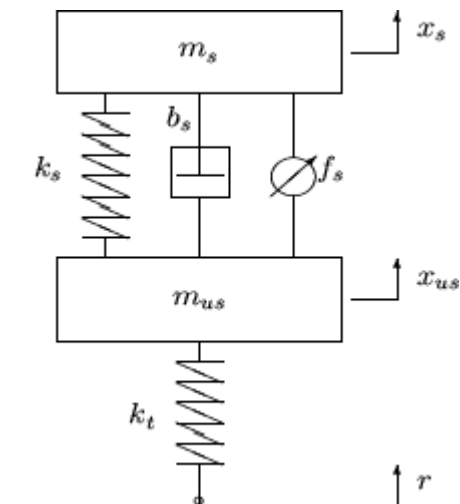
Application of H-Infinity and Mu to Active Suspension Control

Conventional passive suspensions employ a spring and damper between the car body and wheel assembly, representing a tradeoff between conflicting performance metrics such as passenger comfort, road holding, and suspension deflection. Active suspensions allow the designer to balance these objectives using a hydraulic actuator, controlled by feedback, between the chassis and wheel assembly.

In this section, you design an active suspension system for a quarter car body and wheel assembly model using the H_∞ control design technique. You will see the tradeoff between passenger comfort, i.e., minimizing car body travel, versus suspension travel as the performance objective.

Quarter Car Suspension Model

The quarter car model shown is used to design active suspension control laws.



The sprung mass m_s represents the car chassis, while the unsprung mass m_{us} represents the wheel assembly. The spring, k_s , and damper, b_s , represent a passive spring and shock absorber that are placed between the car body and the wheel assembly, while the spring k_t serves to model the compressibility of the pneumatic tire. The variables x_s , x_{us} , and r are the car body travel, the wheel travel, and the road disturbance, respectively. The force f_s , kN, applied between the sprung and unsprung masses, is controlled by feedback and represents the active component of the suspension system. The dynamics of the actuator are ignored in this example, and assume that the control signal is the force f_s . Defining $x_1 := x_s$, $x_2 := \dot{x}_s$, $x_3 := x_{us}$, and $x_4 := \dot{x}_{us}$, the following is the state-space description of the quarter car dynamics.

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{1}{m_s} [k_s (x_1 - x_3) + b_s (x_2 - x_4) - f_s] \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{1}{m_{us}} [k_s (x_1 - x_3) + b_s (x_2 - x_4) - k_t (x_3 - r) - f_s].\end{aligned}$$

The following component values are taken from reference[9].

```
ms = 290;      % kg
mus = 59;     % kg
bs = 1000;    % N/m/s
ks = 16182 ;  % N/m
kt = 190000;  % N/m
```

A linear, time-invariant model of the quarter car model, `qcar`, is constructed from the equations of motion and parameter values. The inputs to the model are the road disturbance and actuator force, respectively, and the outputs are the car body deflection, acceleration, and suspension deflection.

```
A12 = [ 0 1 0 0; [-ks -bs ks bs]/ms ];
A34 = [ 0 0 0 1; [ks bs -ks-kt -bs]/mus];
B12 = [0 0; 0 10000/ms];
B34 = [0 0; [kt -10000]/mus];
C = [1 0 0 0; A12(2,:); 1 0 -1 0; 0 0 0 0];
D = [0 0; B12(2,:); 0 0; 0 1];
qcar = ss([A12; A34], [B12; B34], C, D)
```

It is well known [8] that the acceleration transfer function has an invariant point at the *tirehop frequency*, 56.7 rad/s. Similarly, the suspension deflection transfer function has an invariant point at the *rattlespace frequency*, 23.3 rad/s. The tradeoff between passenger comfort and suspension deflection is because it is not possible to simultaneously keep both transfer functions small around the tirehop frequency and in the low frequency range.

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Linear H-Infinity Controller Design

The design of linear suspension controllers that emphasize either passenger comfort or suspension deflection. The controllers in this section are designed using linear H_∞ synthesis [5]. As is standard in the H_∞ framework, the performance objectives are achieved via minimizing weighted transfer function norms.

Weighting functions serve two purposes in the H_∞ framework: They allow the direct comparison of different performance objectives with the same norm, and they allow for frequency information to be incorporated into the analysis. For more details on H_∞ control design, refer to [4], [6], [7], [11], and [13]. A block diagram of the H_∞ control design interconnection for the active suspension problem is shown below.