# Homework 3 FAQ

*or*

# "There's a slide for that!"

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# Previously on Digital Controls…

- You were given a homework assignment!
- There was a great gnashing of teeth.
- There was a tutorial.
- There was an even greater gnashing of teeth.

# Frequently asked questions

- Here is a list of the most common questions that people have had about the homework
- I have given answers where possible, and hints where appropriate
- Be sure to use the most recent version of the assignment (and slides) from the website

# Question 1

Digitise the following systems, assuming 10 Hz sample rate

i. As difference equations, using Euler's method

ii. ii. As z-domain transfer functions between *x* and *u*

a) $\dot{x} = -3x + u$

b) $\ddot{x} = -2\dot{x} - 4x + \dot{u} + u$

c) $x = 10\left(u + 0.1\int u\,dt + 2\frac{du}{dt}\right)$

d) $\ddot{x} = \dot{y} - 2\dot{x},\ \ddot{y} = -y - \dot{x} + u$

e) $\dot{x} = -\sin(t) - x - \dot{u}$

# Question 1.i

"Do the answers have to be in causal form??"

A. I didn't ask for it, so no. If you *do* put them in causal form, then kudos to you!

# Question 1.i

"In part b, how do I handle the second order derivative??"

$$\ddot{x} = -2\dot{x} - 4x + \dot{u} + u$$

A. That's part of the question. But remember,

$$\ddot{x} = \frac{d}{dt}\dot{x}$$

# Question 1.i

"In part b, how do I handle the *u* derivative??"

$$\ddot{x} = -2\dot{x} - 4x + \dot{u} + u$$

A. That's part of the question. But remember, Euler approximations aren't just for *x*

# Question 1.i

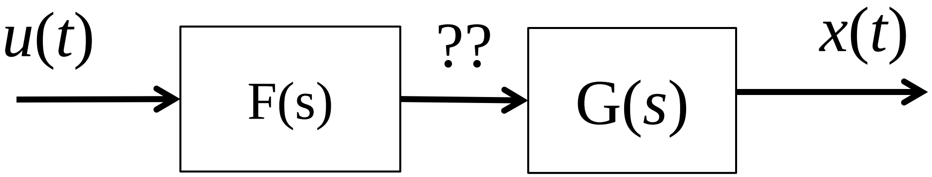
"In part c, do I just differentiate both sides??"

A. Sure, if it works.

# Question 1.i

"In part d, how do I handle *y*??"

A. Remember that linear systems can be represented as a single transfer function between the input and output



# Question 1.i

"In part e, isn't there a constant??"

A. Sure, what about it?

# Question 1.i

"Well, what do we do about it??!"

A. Uh…represent it as a transfer function?

# Question 1.i

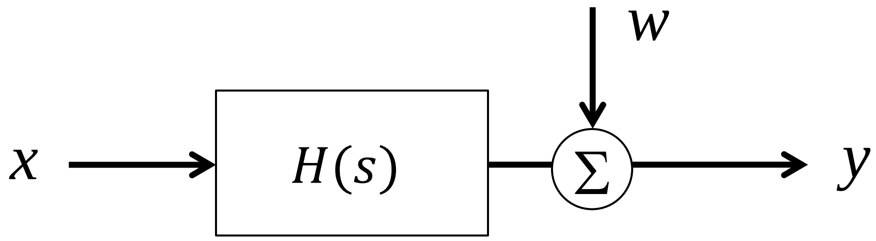
"WTF? You can't represent a constant with a transfer function!!"

A. Er… well, what about load disturbances?

# Question 1.i

"What about load-what-ances?"

A. That's where you have some constant offset applied to the output of your system… you know… like $w(s)$.




Remember *w*? Yeah, that was *awesome*!

# Question 1.i

"So, how do I handle it??"

A. Write the output as a function of the input, with some added extra term. Don't panic.

# Question 1.i

"OMG! I can't find the difference equation of this?? I can find the z-transform, though…"

A. The Laplace, z-transform difference equation representations of a system can all be converted between each other.

Master Pu says:

"*Sometimes the easiest path is not the most direct one.*"

# Question 1.i

"What's with this BS Q&A koan stuff??
Have you been up working at 2:00 am again?"

A. …

Yes.

# Question 1.ii

"Do you want the exact z-transform, or just the approximated version from the difference equation in part i??"

A. Either will do, but I figured since you'd already done the difference equations you might as well just use them.

# Question 2

Find the steady-state value of $x$ at $t \to \infty$ (if any) for each system in part 1

# Question 2

"Do you want the step response or the impulse response??"

A. Either will do. However, in controls (vs signals) it's conventional to use the step response, not the impulse response. Very few physical systems produce impulses. The method I have given you is for step responses.

# Important realisation

*Theorem*: Everything you need to know to answer the homework problems (and exam problems!) is in the lecture slides

*Corollary*: If you're looking up signals books or other (non-control theory) texts or methods, you're probably doing it wrong.

# Question 2

"But my system doesn't go to some steady-state??"

A. Sure, ok.

# Question 3

Sketch the root locus of the system $\ddot{x} = -10x + u$ in feedback with a 5 Hz controller implementing a:

i. lead compensator

ii. lag compensator

iii. PID compensator

Which is more robust to arbitrarily large system gains?

# Question 3

"Do I have to plot the discrete root locus?"

A. Sampled systems are generally understood to be discrete.

# Question 3

"How should I choose the design parameters for these controllers??"

A. Don't worry – I'm not asking for hard numbers, just a generic sketch. I want to see that you understand what each controller looks like in the z-plane and what they do to the root locus.

# Question 4

For what range of proportional feedback gains are the following systems stable?

i. $H(z) = \frac{0.1052}{z-1.105}$

ii. $H(s) = \frac{2}{s^2+4s+4}$, 20 Hz (approximate using Tustin's method)

iii. $H(s) = \frac{2}{s^2+4s+4}$, 20 Hz (exact z-transform)

# Question 4

“’Proportional feedback gain’? Snuh?”

A. That’s not really a question… or even a sentence. Let me try to decipher:

**Proportional**: Where the compensator is multiplier $k$

**Feedback**: Where the output is routed into the input

**Gain**: In this case, the magnitude of $k$

**Snuh**: No idea.

# Question 4

"What the I don't even??"

A. Also not a question. But remember that the stability of a polynomial still holds for the homogenous response:

$$0 = H(s)y$$

# Question 5

Determine analytically the lowest frequency that does not destabilise $H(s)\frac{1}{s^2+s+1}$, given proportional feedback gain of 48.

# Question 5

"What do you want us to do??"

A. Determine analytically the lowest frequency that does not destabilise $H(s) = \frac{1}{s^2+s+1}$, given proportional feedback gain of 48

# Question 5

"Auggh! That's not helpful! How are we supposed to solve this??"

A. Parameterise the characteristic polynomial of $H(z)$ in terms of $T$, and solve for the value of $T$ that makes the absolute magnitude of the poles of $H(z) = 1$.

# Question 5

"Couldn't you just have *asked* for that??"

A. I did. That's what "stability" means.

# Question 6

What design parameter governs the slowest allowable controller sample rate, and why? What design assumptions must be made in order to achieve this slowest rate?

# Question 6

"What the hell?"

A. Hell is a hot, fiery mythological place where wicked souls are sent to burn for all eternity.

# Question 6

"Wha… just tell me the answer already!"

A. I already did. In the lectures. Twice.

You went to the lectures… right?

# Question 6

"No, of course I didn't go to the lectures! Why would I do that??"

A. ...

~sigh~

# Question 6

"I *did* go to your lectures, and I still don't understand. How do I answer this question??"

A. Think carefully about real world systems. All real systems have complex unmodelled high-frequency mechanics, yet digital control (and control in general) still works.

# Question 7

A missile guidance system has a control computer that operates at 50 Hz. The missile's heading is governed by the following dynamic equations:

$$I\ddot{\theta} = -c\dot{\theta} + k\theta + \tau$$

where I = 0.1 is the rotational inertia of the missile, -c = 0.1 is the aerodynamic damping applied by the stabiliser fins, $k$ = 10 is the torque due to nose drag, $\tau$ is the control torque generated by the guide fins, as commanded by the controller. The guide fin actuators are driven by a servomechanism with a two sample communication delay.

Design a controller to regulate the missile's heading, and prove its stability analytically. Write a program that implements your controller in pseudo-code, noting all stored values and constants.

# Question 7

"Sweet Jebus! This system is unstable!?"

A. So?

# Question 7

"What is the input to this system??"

A. The control torque, $\tau$

# Question 7

"How do I handle the delays??"

A. Delays in the z-domain are represented by:

$$x(k-1) = z^{-1}F(z)$$

# Question 7

"What do you mean by 'regulate'??"

A. Maintain a desired value.

# Question 7

"What language do we have to write the program in?"

A. Any, whatever, none – use pseudo-code.

Just show how you'd do it and be sure to include all the essential parts of an actual control algorithm, including inputs, outputs, states, constants, etc.

# General questions

Miscellaneous assignment, course and exam questions

# General questions

"Is this on the exam?"

A. Everything above </assessable> in the notes is *ipso facto* assessable.

# General questions

"Can I use Matlab to answer these questions?"

A. Sure. But you're not going to have Matlab in the exam.

Just say'n...

# General questions

"How are z-plane root loci different from the s-plane??"

A. The mathematics are the same; the roots move in identical ways for identical polynomials. What changes is the physical interpretation of what the location of each pole represents in terms of response behaviour.

# General questions

Q: “Have we *really* been taught everything we need to know to answer these questions??”

A: By this point, yes.

# General questions

“This is *really* hard! Can we have an extension??”

A. Have you had an emergency double splenectomy (or similar)? Have you parents been tragically killed by an escaped circus animal (or similar)? Are you suffering from ebola, black plague, T-virus (or similar)?

# General questions

"Uh… no??"

A. How would you feel if you'd worked hard on an assignment and then someone else got an extension, *just cus'*, but you didn't?

# General questions

"What?! That would be, like, so *unfair*!! Wait… oh… right. Well, can't you just give an extension to *everybody*, then??"

A. I could, but then there'd be zero chance of getting assignments back before the exam. You *really* want to know if you're doing the problems right *before* then.

# General questions

"Wait… so does that mean we'll get the homework back before the exam?"

A. ***No promises***. But we'll try.

OMG THIS!

# General questions

“Are you doing this because you hate us!?”

A. …

Yes.

# Handy hints

- If the question gives a sample period, you probably need to think in discrete space.
  - Note: this is a digital control course, after all.

- Use your intuition in the s-plane (if you have any) to check whether your work in the z-plane makes any sense.

# Handy hints

- The hardest part of these questions is factorising polynomials

- If you can do all the homework problems, you'll be well prepared for the exam.

# Handy hints

- Observe the following:

$$\dot{x} \cong \frac{x(k+1) - x(k)}{T}$$

$$\mathcal{L}\{\dot{x}\} = \mathcal{L}\left\{\frac{x(k+1) - x(k)}{T}\right\}$$

$$sx = \frac{z-1}{T}x$$

$$s^n = \left(\frac{z-1}{T}\right)^n$$

# Handy hints

- In fact!

Forward triangle rule (Euler) $s \rightarrow \frac{z-1}{T}$

Backward triangle rule $s \rightarrow \frac{z-1}{Tz}$

Trapezoid rule (tustin) $s \rightarrow \frac{2}{T}\frac{z-1}{z+1}$

# Tune-in next time for…

**~~~~~~~SPECIAL DOUBLE FEATURE~~~~~~~**

Basics of State-Space Control

*starring*

*Paul Pounds!*

followed by

Estimation and Kalman Filtering

*starring*

*Surya Singh!*

Fun fact: You could always be doing an arts degree