THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

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Family Name	_____________________
First Name	_____________________

School of Information Technology and Electrical Engineering

EXAMINATION

Semester One Final Examinations, 2012

ELEC3004 Signals, Systems and Control

This paper is for St Lucia Campus students.

Examination Duration: 180 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is a Closed Book Examination - Specified materials permitted

During perusal - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Any calculator permitted - unrestricted

One (1) two-sided A4 sheet of handwritten formulae

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

1 x Multiple Choice Answer Sheet

1 x 1cm x 1cm Graph Paper

Rough Paper

Instructions To Students:

For Examiner Use Only

Question	Mark

Total ________

This exam has THREE (3) Sections for a total of 100 Marks

Section 1: Signals & Filters

Please Record Answers On True/False Multiple Choice Answer Sheet
(Total: **20%**, **ALL** questions are of equal value, Please answer ALL)

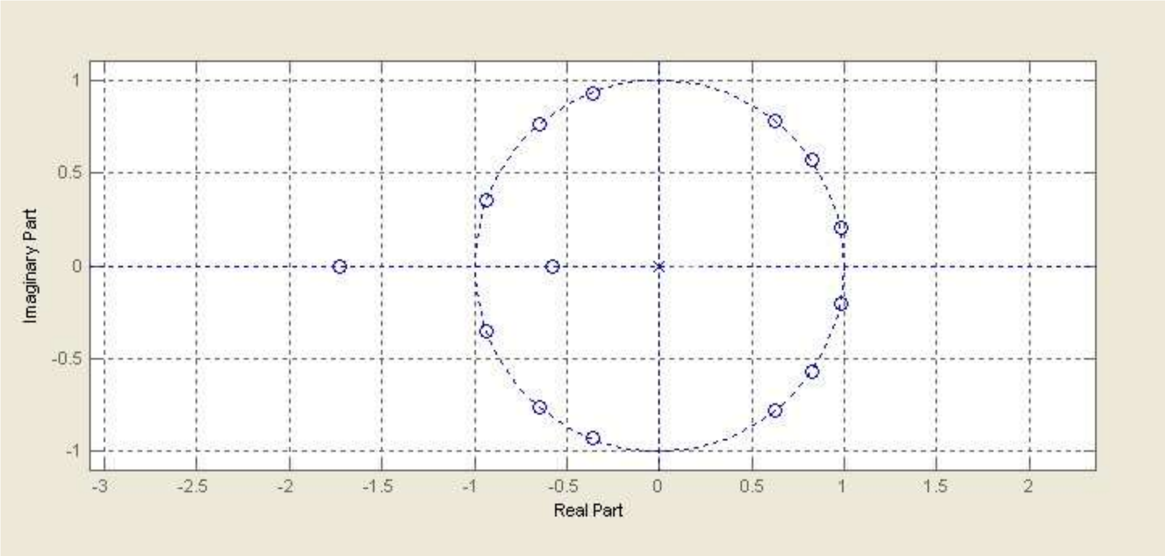


Figure 1 Pole-Zero plot of a filter

1. Consider the pole-zero diagram of the filter of Figure 1. How many taps does this filter have?
   (a) 13
   (b) 14
   (c) 15
   (d) 16
   (e) None of the above

2. Consider the pole-zero diagram of the filter of Figure 1. What type of filter is this?
   (a) Low Pass
   (b) Hilbert Transformer
   (c) Band Pass
   (d) Band Stop
   (e) None of the above

3. Consider the pole-zero diagram of the filter of Figure 1. How many poles does the response have?
   (a) 1
   (b) 7
   (c) 14
   (d) 15
   (e) None of the above

4. Consider the pole-zero diagram of the filter of Figure 1? What is the order of this filter?
   (a) 1
   (b) 7
   (c) 14
   (d) 15
   (e) None of the above

5. Consider the pole-zero diagram of the filter of Figure 1? How many times does the magnitude response go to zero in the stop band?
   (a) 0
   (b) 2
   (c) 4
   (d) 6
   (e) None of the above

6. Consider a zero phase even order lowpass FIR filter designed by the Parks-McClelland Method. Which of the following statements is/are true?
   (i) The impulse response is odd symmetric about the origin
   (ii) The zeros off the unit circle occur in reciprocal conjugate pairs
   (iii) The impulse response at the origin, h(0), is zero

   (a) (i) only
   (b) (ii) only
   (c) (iii) only
   (d) All of the above
   (e) None of the above

7. Which of the following statements is true?
   (a) IIR filters generally have a linear phase response
   (b) FIR filters have low sensitivity to coefficient quantization
   (c) IIR filter architectures are not suitable for high pass filter designs
   (d) All of the above are true
   (e) None of the above is true

8. Which of the following statements is true for a low pass zero phase FIR filter with real impulse response h(n)?
   (i) The impulse response can be anti-symmetric (h(x)=-h(-x))
   (ii) The impulse response can be expressed as a sum of cosine terms
   (iii) Complex valued zeros off the unit circle occur in conjugate reciprocal quads

   (a) Both (i) and (ii)
   (b) Both (ii) and (iii)
   (c) Both (i) and (iii
   (d) All of (i), (ii), and (iii)
   (e) None of (i), (ii) and (iii)

9. Which of the following statements is true for a zero phase FIR filter with real impulse response h(n)?
   (i) The impulse response can be anti-symmetric (h(x)=-h(-x))
   (ii) The impulse response of a low pass filter can be expressed as a sum of cosine terms
   (iii) Complex valued zeros off the unit circle occur in conjugate reciprocal quads

   (a) Both (i) and (ii)
   (b) Both (ii) and (iii)
   (c) Both (i) and (iii
   (d) All of (i), (ii), and (iii)
   (e) None of (i), (ii) and (iii)

10. In the window method of FIR filter design which factor has most influence on the rate of roll-off in the transition band.
    (a) The choice of window
    (b) The number of taps
    (c) Quantization of the filter coefficients
    (d) The bandwidth of the filter
    (e) None of the above

11. In the window method of FIR filter design which factor has most influence on the out of band attenuation.
    (a) The choice of window
    (b) The number of taps
    (c) Quantization of the filter coefficients
    (d) The bandwidth of the filter
    (e) None of the above

Questions 12-14 relate to the following set of specifications for an FIR low pass filter.

Sampling Frequency = 18 kHz
Passband Edge = 250 Hz
Stopband Edge = 350 Hz
Passband Ripple = 1 dB
Stopband Attenuation > 80 dB

12. What is the best estimate of the number of multiplications per second required to process this data with a standard transverse FIR filter (based on the harris formula for filter length)?
    (a) 235,000
    (b) 11.8 million
    (c) 27.4 million
    (d) 38.3 million
    (e) None of the above

13. What is the maximum downsampling factor, M, which could be applied to the output of this filter without significant aliasing distortion?
    (a) 20
    (b) 40
    (c) 60
    (d) 80
    (e) None of the above

14. What is the best estimate of the number of multiplications per second required to process this data with a polyphase downsampling filter?
    (a) 393,000
    (b) 467,000
    (c) 480,000
    (d) 523,000
    (e) None of the above

15. Which is the following is true for the Discrete Fourier Transform (DFT) of a real signal?
    (a) It is purely real and symmetric
    (b) It is purely imaginary and symmetric
    (c) The real part is Even and the imaginary part is Odd
    (d) The real part is Odd and the imaginary part is Even
    (e) None of the above

16. Which of the following window functions has the lowest sidelobe level?
    (a) 4-term Blackman-harris
    (b) Boxcar
    (c) Hann
    (d) Bartlett
    (e) None of the above

17. What is a Hilbert transformer?
    (a) A Japanese toy
    (b) A differentiator
    (c) An integrator
    (d) A wide-band 90 degree phase shifter
    (e) None of the above

18. A DFT transforms N points to N points. What technique do we use if we wish to transform N points to M points where $M<N$? Note that we do not wish to alter the overall shape of the DFT — just change the number of samples.
    (a) Zero packing
    (b) Zero padding
    (c) Wrapping
    (d) Resampling
    (e) None of the above

19. A DFT transforms N points to N points. What technique do we use if we wish to transform N points to M points where $M>N$? Note that we do not wish to alter the overall shape of the DFT — just change the number of samples.
    (a) Zero packing
    (b) Zero padding
    (c) Wrapping
    (d) Resampling
    (e) None of the above

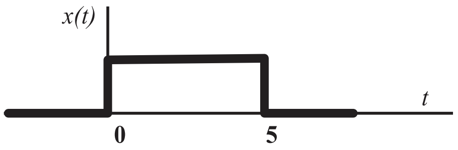
20. Which of the following statements is true?
    (a) FIR filters have numerical stability issues
    (b) IIR filters have numerical stability issues
    (c) Multirate filters are less efficient than single rate filters
    (d) IIR filters do not contain feedback
    (e) None of the above

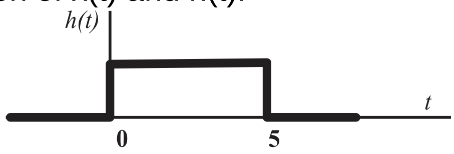
Section 2: Signals & Systems

Please Record Answers in the Answer Book (Total: **20%**)

21. (5%) Convolution

Determine the continuous time convolution of x(t) and h(t):





22. (5%) Signal. Touché!

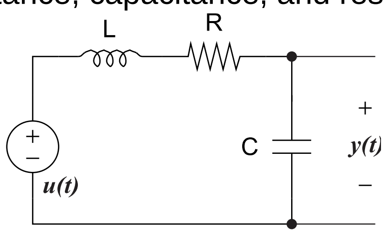
Touché is a Swept Frequency Capacitive Sensing technique that samples the return voltage many times of a capacitive touch surface whose operating frequency is being swept (changed) from a low frequency to a high frequency.

(a) If an AC base voltage applied to the touch surface ranges from 0.5 MHz to 3.5 MHz, what is the Nyquist frequency for sampling this in order to recover the base signal?

(b) If the signal also had high-frequency (2.4 GHz) noise, would the noise appear in the sampled signal? If so, what could be done to prevent this?

23. (10%) Finally, a Reel System Model

A reel of wire can be modelled as follows so as to take into account its (presumably small) inductance, capacitance, and resistance.



(a) Describe this system as a Linear Constant Coefficient ODE.
The input is the driving voltage u(t), and the output is the voltage y(t) at the other end of the wire. (Recall from basic circuit theory that V=IR, V=-LdI/dt, and I=C(dV/dT))

(b) Does this act as a low-pass or a high-pass filter? What is the 3dB cut-off (or corner frequency)?

(c) Suggest two things one could do to physically engineer this reel to have more bandwidth (i.e., shift the corner frequency)?

Section 3: Signals & Digital Control

Please Record Answers in the Answer Book (Total: **60%**)

24. (20%) Consider an electric motor with shaft angular velocity ω. The motor is controlled by input voltage u(t):

    The equations of motion for the system are:

    $$\mathrm{I}\dot{\omega} = \tau$$

    $$\tau = \lambda i$$

    $$u = Ri + \lambda\omega + L\dot{i}$$

    where $\mathrm{I} = 0.01$ Nm/rad/s^2 is the rotational inertia of the rotor, τ is the output torque of the motor in Nm, $\lambda = 0.01 V/_{rads}$ is the flux-linkage coefficient of the motor, ***R = 0.02.*** Ω is the motor internal resistance and ***L*** is the inductance of the motor windings.

    a. Derive a discrete transfer function from ***u(t)*** to ω for this plant, using the matched pole-zero method.
    b. If the inductance is equal to 15 mH, what is the slowest sampling rate that will not destabilise the system under unity-gain proportional negative feedback?
    c. Under what conditions can the inductance be ignored, and the motor treated as a single pole system?

25. (30%) A quadrotor UAV manoeuvers by changing thrusts on opposite pairs of rotors. Consider a simple planar linear model of a quadrotor's pitch angle dynamics:

    $$\mathrm{I}\ddot{\theta} = \tau, \quad \tau = 2k\delta\omega$$

    where θ is the pitch angle, I is the vehicle's rotational inertia, τ is the combined differential rotor torque, ***k*** is a constant, $\delta\omega$ s differential rotor velocity (the control parameter).

    a. Derive a continuous transfer function of the plant, from $\delta\omega$ to θ
    b. Digitise the continuous transfer function using Euler's method.
    c. Plot the transfer function's poles on the Z plane and draw the system root locus under closed loop feedback.
    d. Sketch the positions of the poles and zeros of a PD controller that would stabilise pitch angle under feedback control, and draw the approximate path of the closed loop pole positions with increasing gain.
    e. How does the achievable closed loop system performance change if a pure angular velocity measurement is available in the form $D(z) = \frac{KT_D(z-1)}{T}$?

26. (10%+10% Extra Credit) Political Signals
The despot known as “Singh the Merciless" rules the land with an iron fist.

Four powerful factions support Lord Singh - the Illuminati, the Cryptofascists, the Cheese Hoarders and the People for the Ethical Treatment of Academics. These groups determine their support for Lord Singh's regime based on the daily newspaper they each receive, basing their opinions on other rival factions' reported opinions. Singh will be overthrown if the combined opinions of the Illuminati and the Cryptofascists exceed some very large (but finite) value of dissatisfaction*.

Under Lord Singh's corrupt rule, the daily newspaper has become unreliable and delays are expected. The People for the Ethical Treatment of Academics and the Cheese Hoarders both receive their newspapers a day late. The Illuminati's and Crypofascists' newspapers are not delayed.

After receiving their daily papers:

- The opinion of the Illuminati's will be twice that of the Cheese Hoarders.
- The Cryptofascists opinion will be the same as the People for the Ethical Treatment of Academics'.
- The People for the Ethical Treatment of Academics' opinion will be the negative of the Illuminati's opinion (from yesterday).
- The opinion of the Cheese Hoarders will be the negative of the Cryptofascists' opinion (from yesterday).

**It doesn’t matter how satisfied the constituents ever become - nobody cares that you had a parade in your honour yesterday, if you're deposed and beheaded today.*

a. (10%) Will Singh the Merciless' dire reign last forever?
Prove your answer analytically.

b. (10% extra credit) Using part a, comment on the importance of timely public services/infrastructure to stable governance.

END OF EXAMINATION — Thank you ☺

Please now fold your Graph Paper as a Paper Airplane

Table 1: Commonly used Formulae

The Laplace Transform

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt$$

The $\mathcal{Z}$ Transform

$$F(z) = \sum_{n=0}^{\infty} f[n]z^{-n}$$

IIR Filter Pre-warp

$$\omega_a = \frac{2}{\Delta t}\tan\left(\frac{\omega_d \Delta t}{2}\right)$$

Bi-linear Transform

$$s = \frac{2(1-z^{-1})}{\Delta t(1+z^{-1})}$$

FIR Filter Coefficients

$$c_n = \frac{\Delta t}{\pi}\int_0^{\pi/\Delta t} H_d(\omega)\cos(n\omega\Delta t)\,d\omega$$

Table 2: Comparison of Fourier representations.

Time Domain	**Periodic**	**Non-periodic**	
Discrete	**Discrete Fourier Transform** $\tilde{X}[k] = \frac{1}{N}\sum_{n=0}^{N-1}\tilde{x}[n]e^{-j2\pi kn/N}$ $\tilde{x}[n] = \sum_{k=0}^{N-1}\tilde{X}[k]e^{j2\pi kn/N}$	**Discrete-Time Fourier Transform** $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{j\omega n}\,d\omega$	**Periodic**
Continuous	**Complex Fourier Series** $X[k] = \frac{1}{T}\int_{-T/2}^{T/2}\tilde{x}(t)e^{-j2\pi kt/T}\,dt$ $\tilde{x}(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j2\pi kt/T}$	**Fourier Transform** $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$	**Non-periodic**
	Discrete	**Continuous**	**Freq. Domain**

Table 3: Selected Fourier, Laplace and z-transform pairs.

Signal	$\longleftrightarrow$	Transform	ROC
$\tilde{x}[n] = \sum_{p=-\infty}^{\infty} \delta[n - pN]$	$\xleftrightarrow{DFT}$	$\tilde{X}[k] = \frac{1}{N}$	
$x[n] = \delta[n]$	$\xleftrightarrow{DTFT}$	$X(e^{j\omega}) = 1$	
$\tilde{x}(t) = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$\xleftrightarrow{FS}$	$X[k] = \frac{1}{T}$	
$\delta_T[t] = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$\xleftrightarrow{FT}$	$X(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$	
$\cos(\omega_0 t)$	$\xleftrightarrow{FT}$	$X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$	
$\sin(\omega_0 t)$	$\xleftrightarrow{FT}$	$X(j\omega) = j\pi\delta(\omega + \omega_0) - j\pi\delta(\omega - \omega_0)$	
$x(t) = \begin{cases} 1 & \text{when } \lvert t\rvert \leqslant T_0, \\ 0 & \text{otherwise.} \end{cases}$	$\xleftrightarrow{FT}$	$X(j\omega) = \frac{2\sin(\omega T_0)}{\omega}$	
$x(t) = \frac{1}{\pi t}\sin(\omega_c t)$	$\xleftrightarrow{FT}$	$X(j\omega) = \begin{cases} 1 & \text{when } \lvert\omega\rvert \leqslant \lvert\omega_c\rvert, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\xleftrightarrow{FT}$	$X(j\omega) = 1$	
$x(t) = \delta(t - t_0)$	$\xleftrightarrow{FT}$	$X(j\omega) = e^{-j\omega t_0}$	
$x(t) = u(t)$	$\xleftrightarrow{FT}$	$X(j\omega) = \pi\delta(w) + \frac{1}{jw}$	
$x[n] = \frac{\omega_c}{\pi} \operatorname{sinc} \omega_c n$	$\xleftrightarrow{DTFT}$	$X(e^{j\omega}) = \begin{cases} 1 & \text{when } \lvert\omega\rvert < \lvert\omega_c\rvert, \\ 0 & \text{otherwise.} \end{cases}$	
$x(t) = \delta(t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = 1$	all s
(unit step) $x(t) = u(t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{1}{s}$	
(unit ramp) $x(t) = t$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{1}{s^2}$	
$x(t) = \sin(s_0 t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{s_0}{(s^2 + {s_0}^2)}$	
$x(t) = \cos(s_0 t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{s}{(s^2 + {s_0}^2)}$	
$x(t) = e^{s_0 t}u(t)$	$\xleftrightarrow{\mathcal{L}}$	$X(s) = \frac{1}{s - s_0}$	$\mathfrak{Re}\{s\} > \mathfrak{Re}\{s_0\}$
$x[n] = \delta[n]$	$\xleftrightarrow{z}$	$X(z) = 1$	all z
$x[n] = \delta[n - m]$	$\xleftrightarrow{z}$	$X(z) = z^{-m}$	
$x[n] = u[n]$	$\xleftrightarrow{z}$	$X(z) = \frac{z}{z - 1}$	
$x[n] = z_0^n u[n]$	$\xleftrightarrow{z}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$\lvert z\rvert > \lvert z_0\rvert$
$x[n] = -z_0^n u[-n - 1]$	$\xleftrightarrow{z}$	$X(z) = \frac{1}{1 - z_0 z^{-1}}$	$\lvert z\rvert < \lvert z_0\rvert$
$x[n] = a^n u[n]$	$\xleftrightarrow{z}$	$X(z) = \frac{z}{z - a}$	$\lvert z\rvert < \lvert a\rvert$

Table 4: Properties of the Discrete-time Fourier Transform.

Property	Time domain	Frequency domain
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
Differentiation (frequency)	$nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
Time-shift	$x[n-n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Frequency-shift	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Convolution	$x_1[n] * x_2[n]$	$X_1(e^{j\omega})X_2(e^{j\omega})$
Modulation	$x_1[n]x_2[n]$	$\frac{1}{2\pi}X_1(e^{j\omega}) \circledast X_2(e^{j\omega})$
Time-reversal	$x[-n]$	$X(e^{-j\omega})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(e^{j\omega}) = X^*(e^{-j\omega})$
Symmetry (imag)	$\mathfrak{Re}\{x[n]\} = 0$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$
Parseval	$\displaystyle\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\lvert X(e^{j\omega})\right\rvert^2 d\omega$	

Table 5: Properties of the Fourier series.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1[k] + bX_2[k]$
Differentiation (time)	$\dfrac{d\tilde{x}(t)}{dt}$	$\dfrac{j2\pi k}{T}X[k]$
Time-shift	$\tilde{x}(t-t_0)$	$e^{-j2\pi k t_0/T}X[k]$
Frequency-shift	$e^{j2\pi k_0 t/T}\tilde{x}(t)$	$X[k-k_0]$
Convolution	$\tilde{x}_1(t) \circledast \tilde{x}_2(t)$	$TX_1[k]X_2[k]$
Modulation	$\tilde{x}_1(t)\tilde{x}_2(t)$	$X_1[k] * X_2[k]$
Time-reversal	$\tilde{x}(-t)$	$X[-k]$
Conjugation	$\tilde{x}^*(t)$	$X^*[-k]$
Symmetry (real)	$\mathfrak{Im}\{\tilde{x}(t)\} = 0$	$X[k] = X^*[-k]$
Symmetry (imag)	$\mathfrak{Re}\{\tilde{x}(t)\} = 0$	$X[k] = -X^*[-k]$
Parseval	$\displaystyle\frac{1}{T}\int_{-T/2}^{T/2} \lvert\tilde{x}(t)\rvert^2 dt = \sum_{k=-\infty}^{\infty} \lvert X[k]\rvert^2$	

Table 6: Properties of the Fourier transform.

Property	Time domain	Frequency domain
Linearity	$a\tilde{x}_1(t) + b\tilde{x}_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Duality	$X(jt)$	$2\pi x(-\omega)$
Differentiation	$\frac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$
Time-shift	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency-shift	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega)X_2(j\omega)$
Modulation	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(j\omega) * X_2(j\omega)$
Time-reversal	$x(-t)$	$X(-j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetry (real)	$\mathfrak{Im}\{x(t)\} = 0$	$X(j\omega) = X^*(-j\omega)$
Symmetry (imag)	$\mathfrak{Re}\{x(t)\} = 0$	$X(j\omega) = -X^*(-j\omega)$
Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
Parseval	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\, d\omega$	

Table 7: Properties of the z-transform.

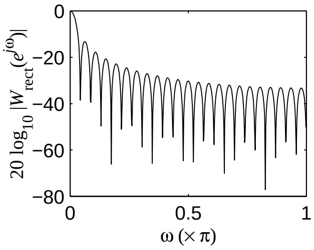
Property	Time domain	z-domain	ROC
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Time-shift	$x[n - n_0]$	$z^{-n_0}X(z)$	$R_x^\dagger$
Scaling in z	$z_0^n x[n]$	$X(z/z_0)$	$\lvert z_0\rvert R_x$
Differentiation in z	$nx[n]$	$-z\frac{dX(z)}{dz}$	$R_x^\dagger$
Time-reversal	$x[-n]$	$X(1/z)$	$1/R_x$
Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
Symmetry (real)	$\mathfrak{Im}\{x[n]\} = 0$	$X(z) = X^*(z^*)$	
Symmetry (imag)	$\mathfrak{Re}\{X[n]\} = 0$	$X(z) = -X^*(z^*)$	
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$\subseteq R_{x_1} \cap R_{x_2}$
Initial value	$x[n] = 0, n < 0 \Rightarrow x[0] = \lim_{z\to\infty} X(z)$		

† $z = 0$ or $z = \infty$ may have been added or removed from the ROC.

Table 8: Commonly used window functions.

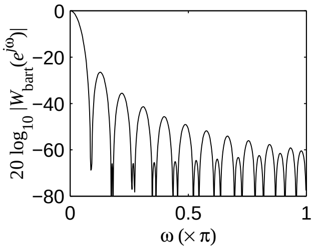
Rectangular:

$$w_{\text{rect}}[n] = \begin{cases} 1 & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



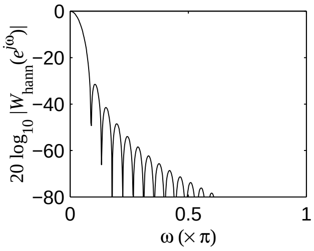
Bartlett (triangular):

$$w_{\text{bart}}[n] = \begin{cases} 2n/M & \text{when } 0 \leqslant n \leqslant M/2, \\ 2 - 2n/M & \text{when } M/2 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



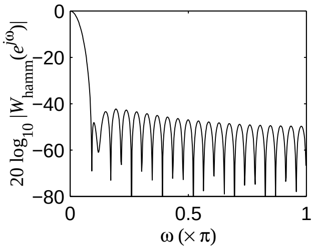
Hanning:

$$w_{\text{hann}}[n] = \begin{cases} \frac{1}{2} - \frac{1}{2}\cos(2\pi n/M) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



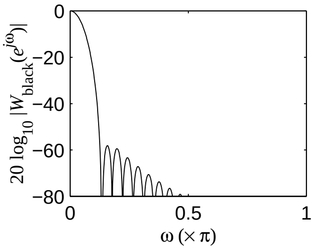
Hamming:

$$w_{\text{hamm}}[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



Blackman:

$$w_{\text{black}}[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) & \\ \quad + 0.08\cos(4\pi n/M) & \text{when } 0 \leqslant n \leqslant M, \\ 0 & \text{otherwise.} \end{cases}$$



Type of Window	Peak Side-Lobe Amplitude (Relative; dB)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)
Rectangular	-13	$4\pi/(M+1)$	-21
Bartlett	-25	$8\pi/M$	-25
Hanning	-31	$8\pi/M$	-44
Hamming	-41	$8\pi/M$	-53
Blackman	-57	$12\pi/M$	-74