Convolution

ELEC 3004: Signals, Systems & Control

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http://courses.itee.uq.edu.au/elec3004/2012s1/

Schedule of Events

Week	Date	Lecture Title
1	1-Mar	Overview
2	5-Mar	Signals & Systems
	8-Mar	Sampling
3	12-Mar	Laplace
	15-Mar	LTI
4	19-Mar	Convolution and Discrete Fourier Series
	22-Mar	Fourier Transform
5	26-Mar	Fourier Transform Operations
	29-Mar	Applications: DFFT and DCT
6	2-Apr	Exam 1 (10%)
	5-Apr	(Guest Lecture from Industry)
7	16-Apr	Data Acquisition & Interpolation
	19-Apr	Noise
8	23-Apr	Filters & IIR Filters
	26-Apr	FIR Filters
9	30-Apr	Multirate Filters
	3-May	Filter Selection
10	7-May	Holiday
	10-May	Quiz (10%)
11	14-May	z-Transform
	17-May	Introduction to Digital Control
12	21-May	Stability of Digital Systems
	24-May	Estimation
13	28-May	Kalman Filters & GPS
	31-May	Applications in Industry

Convolution Systems

- Convolution system with input u (u(t) = 0, t < 0) and output y:

$$y(t) = \int_0^t h(\tau)u(t-\tau)\,d\tau = \int_0^t h(t-\tau)u(\tau)\,d\tau$$

- abbreviated:

$$y = h * u$$

- in the frequency domain:

$$Y(s) = H(s)U(s)$$

Properties

1. convolution systems are linear:

$$h * (\alpha u_1 + \beta u_2) = \alpha(h * u_1) + \beta(h * u_2)$$

2. convolution systems are causal: the output y(t) at time t depends only on past inputs
3. convolution systems are time-invariant (if we shift the signal, the output similarly shifts)

➔

$$\tilde{u}(t) = \begin{cases} 0 & t < T \\ u(t-T) & t \geq 0 \end{cases}$$

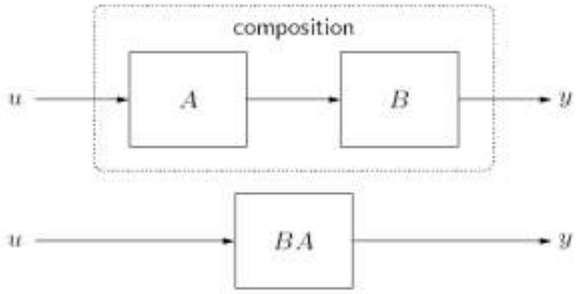
$$\tilde{y}(t) = \begin{cases} 0 & t < T \\ y(t-T) & t \geq 0 \end{cases}$$

Properties (II)

Composition of convolution systems corresponds to:

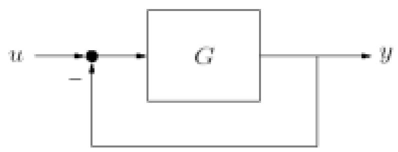
– multiplication of transfer functions

– convolution of impulse responses



- Thus:
  – We can manipulate block diagrams with transfer functions as if they were simple gains
  – convolution systems commute with each other

Feedback Connection



- In the time domain:

$$y(t) = \int_0^t g(t-\tau)(u(\tau) - y(\tau))\, d\tau$$

- In the frequency domain:
  – Y=G(U-Y)

  ➔Y(s) = H(s)U(s)

$$H(s) = \frac{G(s)}{1 + G(s)}$$