## LTI

# ELEC 3004: Signals, Systems & Control

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Lecture # 5 March 15, 2012

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http://courses.itee.uq.edu.au/elec3004/2012s1/



## Schedule of Events

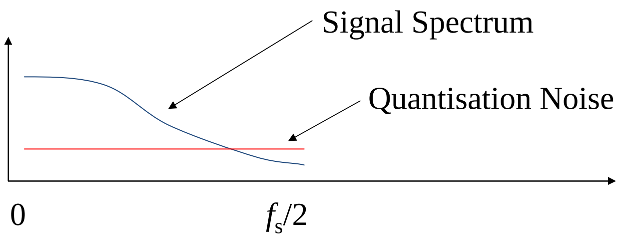
| Week | Date | Lecture Title |
|---|---|---|
| 1 | 1-Mar | Overview |
| 2 | 5-Mar | Signals & Systems |
| | 8-Mar | Sampling |
| 3 | 12-Mar | Laplace |
| | 15-Mar | LTI |
| 4 | 19-Mar | Convolution and Discrete Fourier Series |
| | 22-Mar | Fourier Transform |
| 5 | 26-Mar | Fourier Transform Operations |
| | 29-Mar | Applications: DFFT and DCT |
| 6 | 2-Apr | Exam 1 (10%) |
| | 5-Apr | (Guest Lecture from Industry) |
| 7 | 16-Apr | Data Acquisition & Interpolation |
| | 19-Apr | Noise |
| 8 | 23-Apr | Filters & IIR Filters |
| | 26-Apr | FIR Filters |
| 9 | 30-Apr | Multirate Filters |
| | 3-May | Filter Selection |
| 10 | 7-May | Holiday |
| | 10-May | Quiz (10%) |
| 11 | 14-May | z-Transform |
| | 17-May | Introduction to Digital Control |
| 12 | 21-May | Stability of Digital Systems |
| | 24-May | Estimation |
| 13 | 28-May | Kalman Filters & GPS |
| | 31-May | Applications in Industry |

## Annoucements

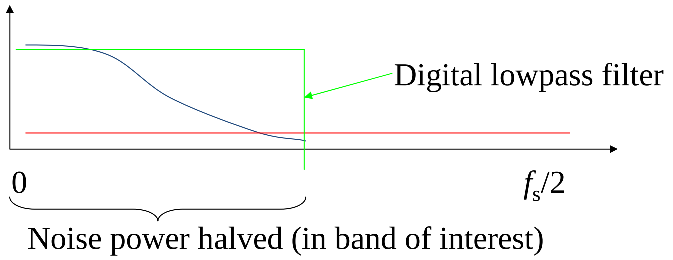
- Lab 1 Repeats
- Course Profile Says: Lab 1 + Lab 2 = 10%
  SO: Lab 1 = 5% (sorry!)
- Homework 1 is posted – still due March 23
- Prof. Lovell can not come on Monday.
  So, we'll talk about convolution then and FFT on Thursday.



## Whoops 2: Quantization and Freq.




Doubled Sampling Frequency



## ODE's and Linear Systems

- Linear system described by differential equation

$$a_0 y + a_1 \frac{dy}{dt} + \cdots + a_n \frac{d^n y}{dt^n} = b_0 x + b_1 \frac{dx}{dt} + \cdots + b_m \frac{d^m x}{dt^m}$$

- Which using Laplace Transforms can be written as

$$a_0 Y(s) + a_1 s Y(s) + \cdots + a_n s^n Y(s) = b_0 X(s) + b_1 s X(s) + \cdots + b_m s^m X(s)$$
$$A(s) Y(s) = B(s) X(s)$$

where *A(s)* and *B(s)* are polynomials in *s*



## Transfer Function

- Transfer function can be written as

$$H(s) = \frac{Y(s)}{X(s)} = \frac{B(s)}{A(s)}$$
$$= \frac{b_0 + b_1 s + \cdots + b_m s^m}{a_0 + a_1 s + \cdots + a_n s^n}$$

- Transfer functions have:
  - **Poles**
    - Infinite value of *H(s)*, i.e., when *A(s)* = 0 (roots of *A(s)*)
  - **Zeros**
    - Zeros value of *H(s)*, i.e., when *B(s)* = 0 (roots of *B(s)*)

The poles & zeros of H(s) define freq response & stability

## Poles & zeros: Example

- Transfer function

$$H(s) = \frac{s^2 + 2s + 2}{s^2 + 4s + 13}$$

- Zeros at $s^2 + 2s + 2 = 0$

$$s = \frac{-2 \pm \sqrt{2^2 - 4(2)}}{2} = -1 \pm j.$$

- Poles at

$$s^2 + 4s + 13 = 0$$

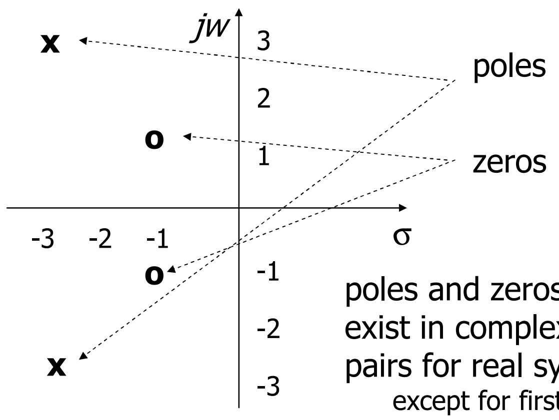
$$s = -2 \pm j3$$

Poles/zero plot drawn on complex *s*-plane



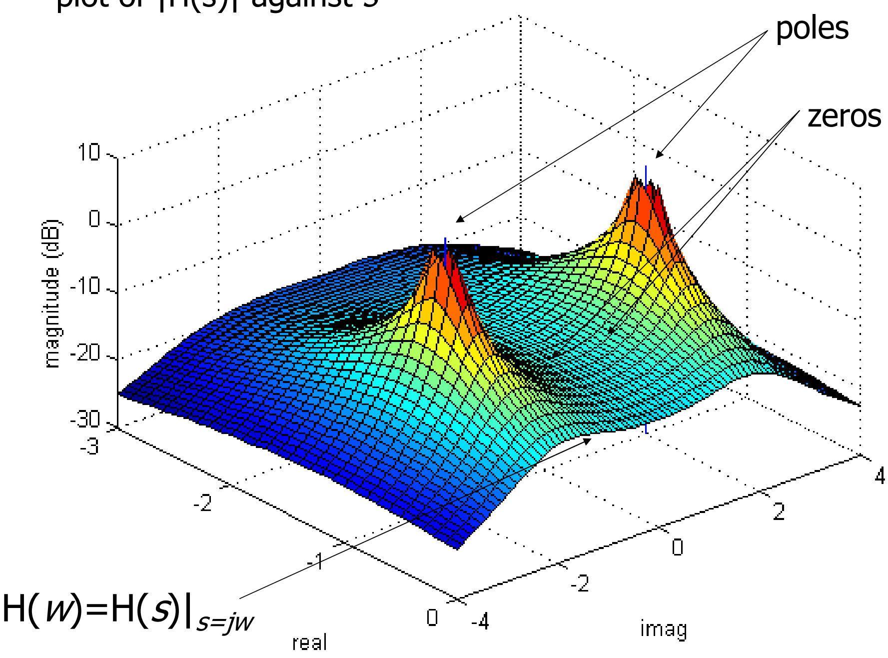
## Pole zero plot

*s*-plane




poles and zeros always exist in complex conjugate pairs for real systems

except for first order systems when they occur on the σ axis

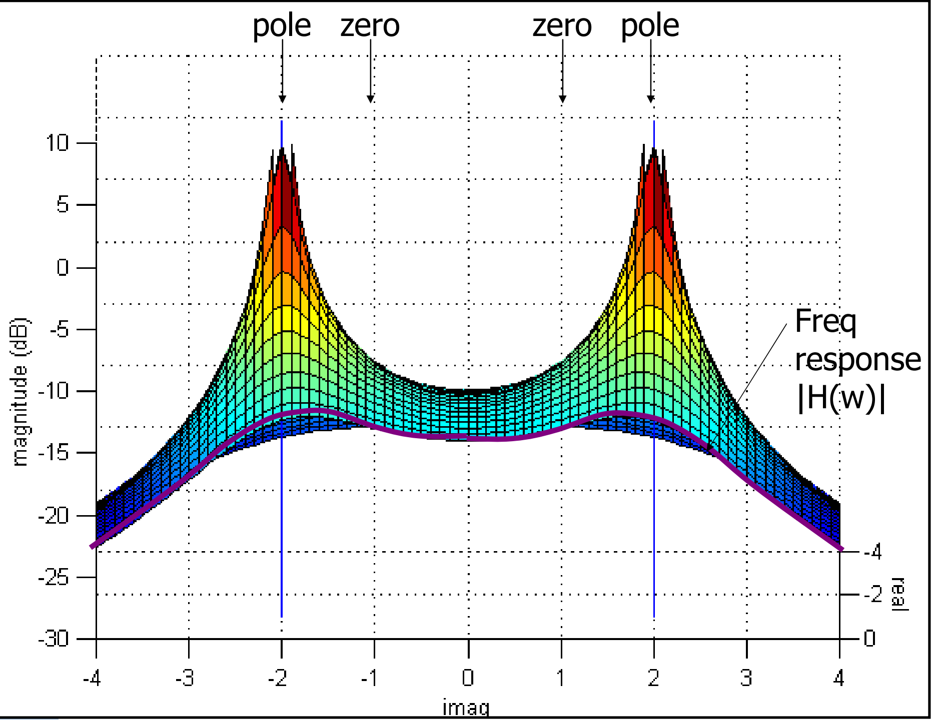



## Transfer Function and Frequency Response

- Frequency response:
  - similar to transfer function
- Provided H(s) has poles only in the left half of the *s*-plane
  - substitute *jw* for *s* (i.e., σ = 0)
  - remember s = σ + *jw*

$$H(w) = \frac{Y(w)}{X(w)}$$

$$H(s) = \frac{1}{s+7}; H(w) = \frac{1}{jw+7}$$




# Frequency response

- Transfer function and frequency response can be written in terms of poles and zeros

$$H(s) = \frac{A(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$$H(w) = \frac{A(jw-z_1)(jw-z_2)\cdots(jw-z_m)}{(jw-p_1)(jw-p_2)\cdots(jw-p_n)}$$

$$|H(w)| = \frac{A|jw-z_1||jw-z_2|\cdots|jw-z_m|}{|jw-p_1||jw-p_2|\cdots|jw-p_n|}$$ ← Magnitude

$$\angle H(w) = \angle(jw-z_1) + \angle(jw-z_2) + \cdots + \angle(jw-z_m) - \angle(jw-p_1) - \angle(jw-p_2) - \cdots - \angle(jw-p_n)$$ ← Phase

## Frequency Response Summary

- H(w) = H(s)$|_{s=jw}$ i.e., imaginary axis
  - gain: vector length from pole/zero to $w_0$
  - Phase: horizontal angle from pole/zero to $w_0$
- General case (multiple pole/zeros)
  - gain: product of vector lengths of all poles/zeros
  - Phase: sum angles of all vectors
- Poles: zero in denominator
  - Peak in frequency response |H(w)| at $\Im(p)$
    - Height of peak is inversely proportional to $|\Re(p)|$
- Zeros: zero in numerator
  - Minimum (zero) in |H(w)| at $\Im(p)$
    - Depth of minimum inversely proportional to $|\Re(p)|$



## Real Systems

- have poles and zeros

$$H(s) = \frac{A(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

*m* zeros
*n* poles
where *m* < *n*
H(s) is proper fraction

Partial Fraction Expansion gives,

$$H(s) = \frac{A_1}{(s-p_1)} + \frac{A_2}{(s-p_2)} + \cdots + \frac{A_n}{(s-p_n)}$$

Taking inverse Laplace Transform gives,

$$h(t) = A_1 \exp(p_1 t) + A_2 \exp(p_2 t) + \cdots + A_n \exp(p_n t)$$

Impulse response is sum of complex phasors which depend on location of poles in *s*-plane

## Real Signals

- Can be modelled as weighted sums of impulses (remember?)

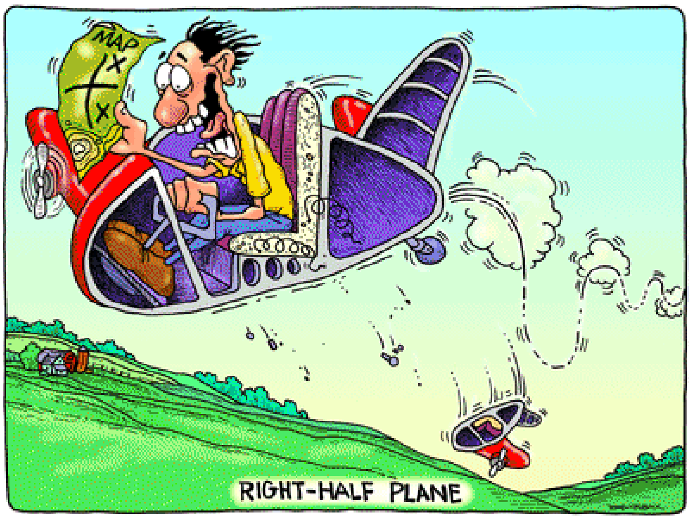
- Therefore, if impulse response is made up of decaying complex exponentials
  - i.e., is absolutely integrable

- Then any bounded input will produce a bounded input
  - i.e., system is stable (BIBO)



## System Stability

- *A necessary and sufficient condition that a system is stable is that all poles of the transfer function lie in the left half of the s-plane, i.e., have negative real parts.*






# Fourier and Laplace

- Fourier Series
  - Developed first, easiest to understand
  - But only applies to periodic signals (e.g., in labs)
- Fourier Transform
  - Extension of Fourier series to non-periodic signals
  - Generally used to analyse **signals**
  - Also frequency response H(w) of **systems**
- Laplace Transform
  - generalisation of Fourier transform ($s = \sigma + jw$)
  - generally used to analyse/design **systems** i.e., H(s)
    - e.g., control systems, filters
  - H(s) tells us about H(w) and stability of system

## Hitchhikers Guide: continuous time