Types of Filters

- These types of filters are easy to design with the standard remez algorithm
  – Low Pass, High Pass, Band Pass, Band Stop,Multiband, Arbitrary Magnitude
  – In Matlab may need to use the ‘hilbert’ switch for some filters (e.g. even length high pass, hpremez.m)
- The following are easy to design with a slight modification to the algorithm. These filters generally have zero response near DC and Fs/2.
  – Differentiator $H(f) = j2\pi f$
  – Hilbert Transform, $H(f) = -j\,\mathrm{sgn}(f)$

Differentiator

- Differentiation corresponds to a filter $H(f)=j2\pi f$.
- The inverse transform of this filter is

$$h(n) = (-1)^n \frac{1}{n}, n \neq 0$$

- The first three terms of this response are (-1,0,1) which is the standard approximation for differentiation learnt in calculus. (central finite difference)

The spectral response of this truncated filter is

$$H(z) = z^{+1} - z^{-1}$$

$$H(f) = e^{+j2\pi f} - e^{-j2\pi f}$$

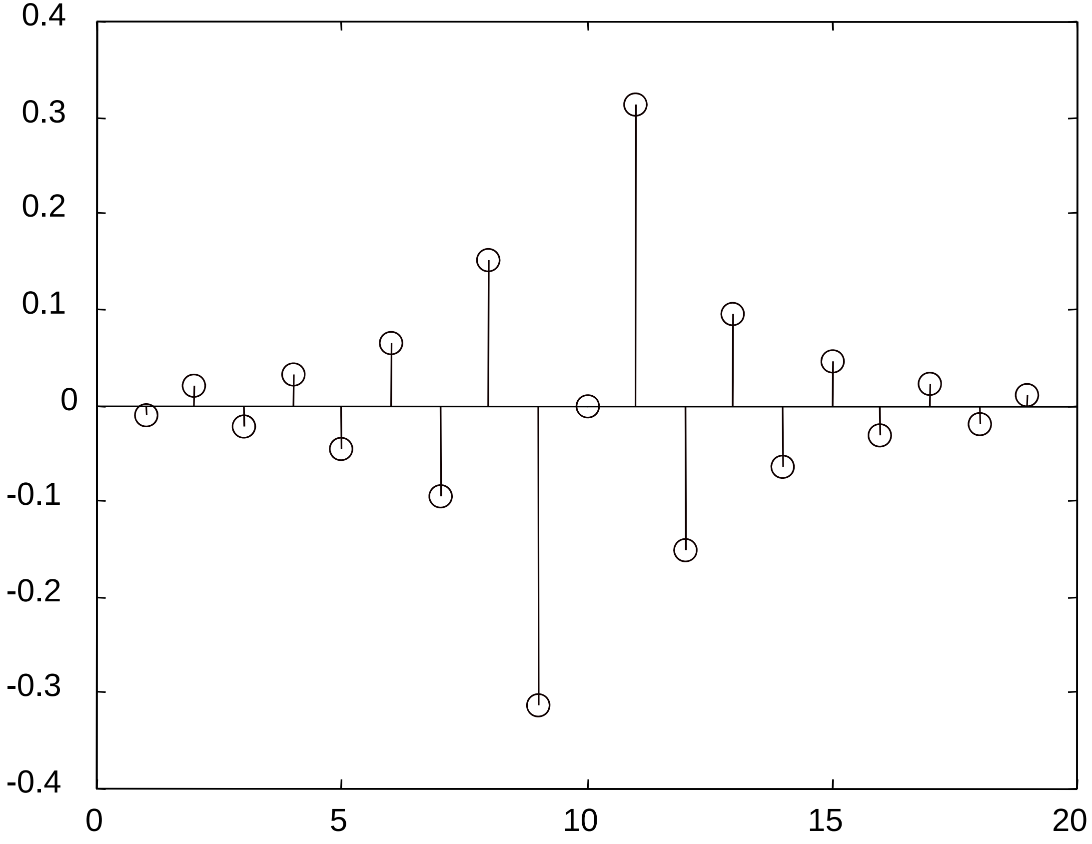
$$= 2j\sin(2\pi f)$$

which for small *f* is approximately

$$H(f) \cong j4\pi f$$

In the design we generally specify a bandwidth over which the filter acts as a differentiator.

Impulse Response of Differentiator



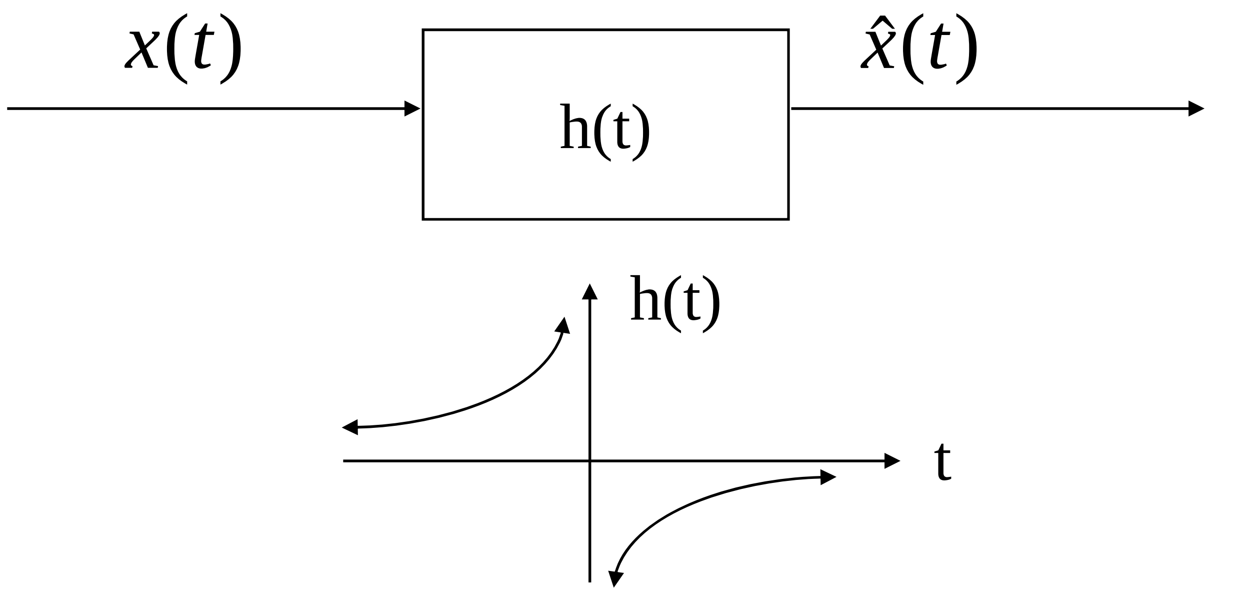
Hilbert Transform

- A Hilbert Transform is used to form the Analytic Signal. This signal is a complex signal with the property that its FT has only positive (or negative) frequencies. [e.g., the analytic signal corresponding to $\cos(2\pi ft)$ is $\exp(j2\pi ft)$]
- This signal is classically used in Single Sideband Modulation.
- The Hilbert Transform is defined by

$$\hat{h}(t) = \int_{-\infty}^{+\infty} \frac{h(\tau)}{t-\tau} d\tau$$

Hilbert Transform

What does this mean? This is the instruction to perform a convolution of the input signal h(t) with a filter response 1/t.



Hilbert Transform

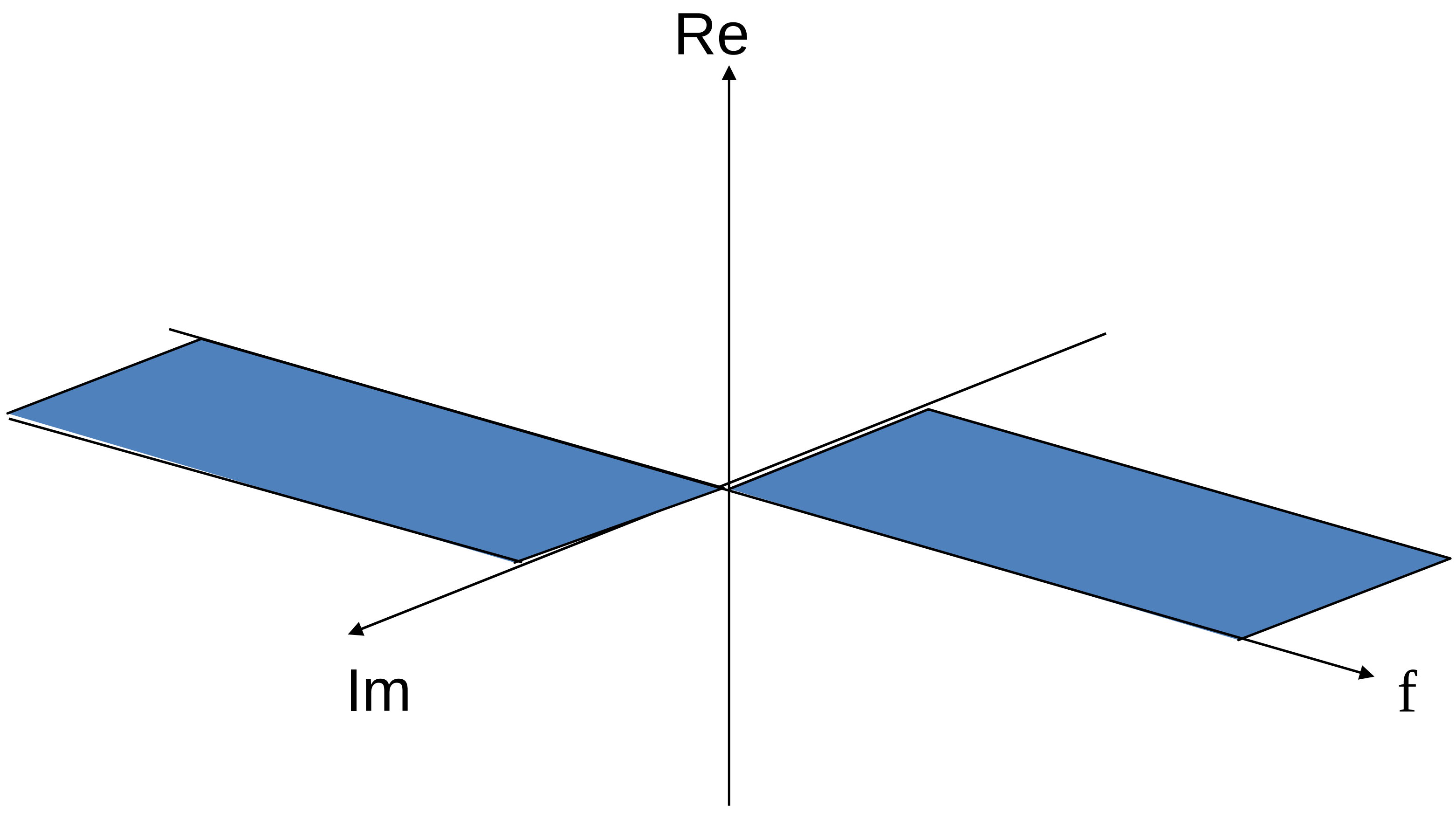
We can form the HT by designing a filter with the frequency response corresponding to the impulse response 1/t. This response is

$$h(f) = -j\,\mathrm{sgn}(f)$$

This frequency response describes a wide-band 90 degrees phase shifter...that is positive frequencies are shifted -90 degrees and negative frequencies are shifted +90 degrees.

Note: HT of $\sin(2\pi ft)$ is $\cos(2\pi ft)$

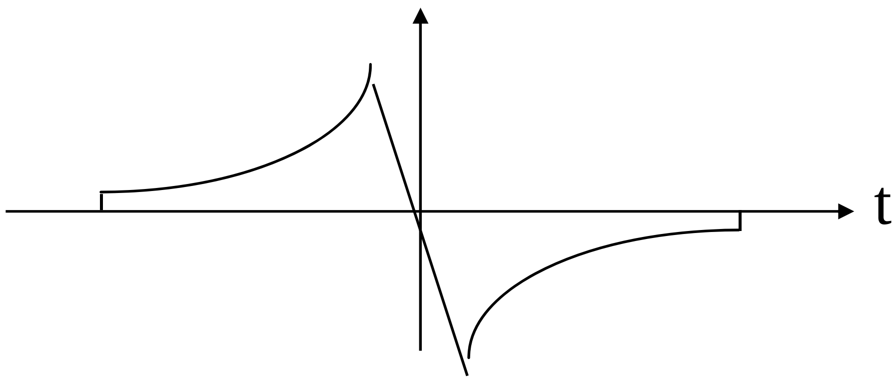
Hilbert Transform



This is a terrible filter! The impulse response has a bad singularity at the origin. Further, the function extends to $\pm\infty$ on the time line.

To realize this filter we need to modify the response

- Truncate to make finite
- Change behaviour at origin, affects low freq performance



Discrete Time HT

For discrete time implementation, the spectrum must be periodic in the sample rate. The form of the sampled data spectrum is seen to be

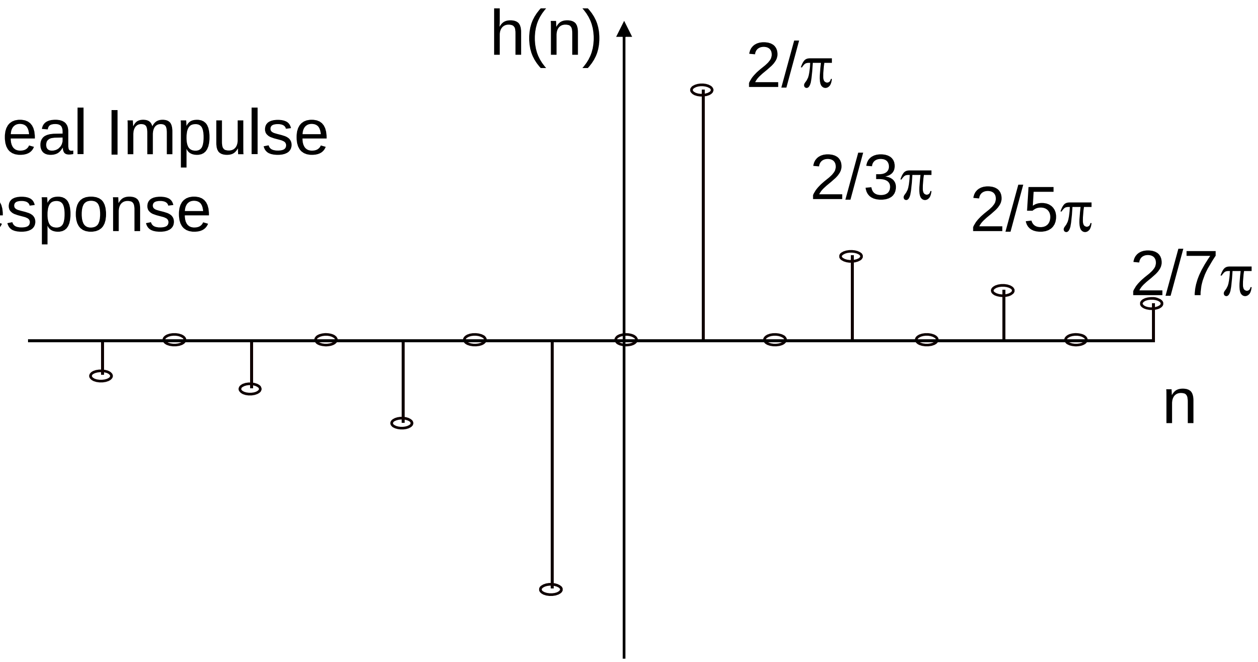
$$H(f) = -j, 0 < f < f_s / 2$$
$$+j, -f_s / 2 < f < 0$$

The impulse response is

$$h(n) = \frac{2}{\pi} \sum_{-\infty}^{+\infty} \frac{1}{n} \; \forall \text{ odd} \quad n$$

Discrete Time HT

Ideal Impulse response



Other Effects

- Coefficient Quantization in Finite Precision
  - As a rough guide the maximum stopband attenuation must be about 5*n dB, where n is the number of bits. For example the noise floor is 80 dB with 16 bit arithmetic.
  - Can optimise filter design taking into account finite precision by performing an exhaustive search around desired function. There are algorithms available for this but the computational load becomes prohibitive for N greater than about 40.