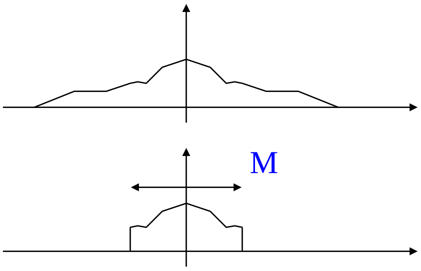
Parks-McClellan Method

- Often called the Remez exchange method.
- This method designs an optimal linear phase filter directly from the design specifications.
- This is the standard method for FIR filter design.
- In matlab, this method is available as remez().

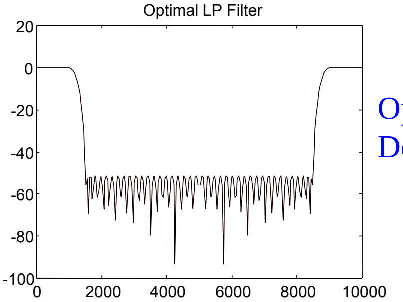
Approximation Errors

- From the theory of the Fourier series, the rectangular window design method (truncation of the impulse response) gives the best mean square (L^2) approximation to a desired frequency response for a given filter length M.



Minimax Design

- However simple truncation leads to adverse behaviour near discontinuities and in the stop band.
- Better filters generally result from minimization of the maximum error (L_∞) or a frequency weighed error criterion.



Optimal Filter Design

Optimal Filters

Consider a linear phase FIR filter for which h[n] = h[-n]

Take Fourier Transforms

$$A(e^{j\omega}) = \sum_{n=-L}^{L} h[n]e^{-j\omega n}, \omega = 2\pi f \ / \ f_s$$

with $L = M / 2$

$$A(e^{j\omega}) = h[0] + \sum_{n=1}^{L} 2h[n]\cos(\omega n)$$

Now the terms $\cos(\omega n)$ can be expressed as polynomials of degree n in $\cos\omega$

Chebyshev Polynomials

$\cos \omega n = T_n(\cos \omega)$

where $T_n(x)$ is the nth order Chebyshev polynomial of the first kind defined by the recursion

$T_0(x) = 1\,, T_1(x) = x$

$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$

for example

$T_2(x) = 2x^2 - 1;\; T_3(x) = 4x^3 - 3x$

Thus A can be rewritten as a polynomial of degree L in cos ω.

$$A(e^{j\omega}) = \sum_{k=0}^{L} a_k (\cos\omega)^k$$

If we replace cos ω with x we obtain

$$A(e^{j\omega}) = P(x)\big|_{x=\cos\omega}$$

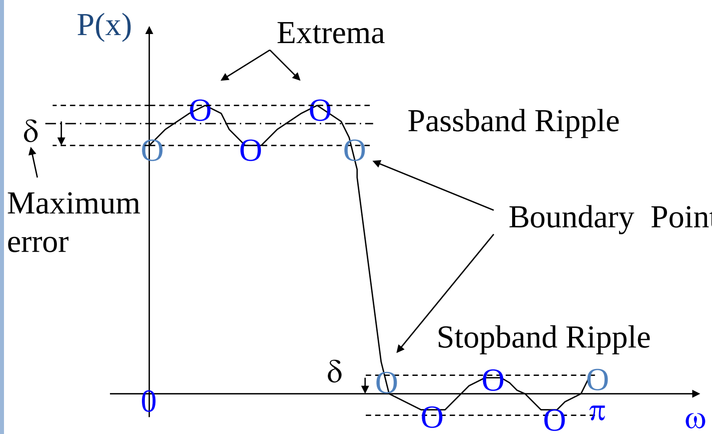
where $P(x)$ is the Lth order polynomial

$$P(x) = \sum_{k=0}^{L} a_k x^k$$

Optimal Filters

- **Alternation Theorem** : The polynomial of degree L that minimises the maximum error will have at least L+2 extrema. The optimal frequency response will just "kiss" the maximum ripple bounds
- Extrema must occur at the pass and stop band edges and at either $\omega=0$ or π or both.
- Now the derivative of a polynomial of degree L is a polynomial of degree L-1, which can be zero in at most L-1 places. So the maximum number of local extrema is the L-1 local extrema plus the 4 band edges. That is L+3.

Optimal Response



L=7th order polynomial, number of extrema = 10

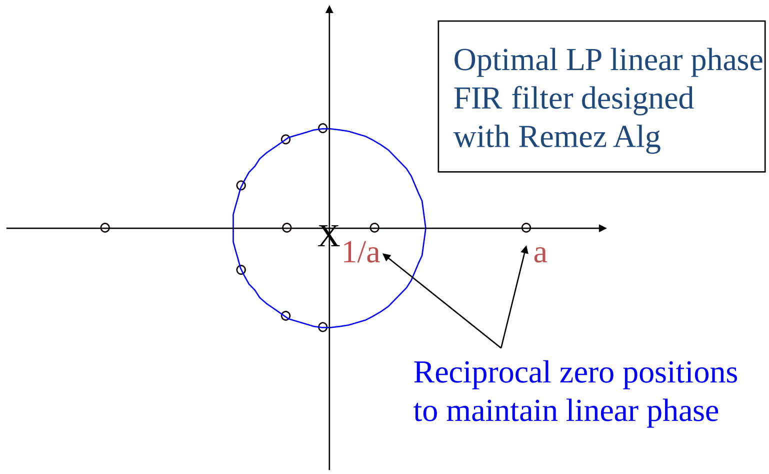
The Method

- We know where the boundary points are from the band edge specifications. At least 3 of these points must be extrema.
- We know how many local extrema there are from the estimated filter length (harris formula or similar) but we don't know their positions.
- Guess the positions of the extrema are evenly spaced in the pass and stop bands.
- Perform polynomial interpolation and reestimate positions of local extrema.
- Move extrema to new positions and iterate until the extrema stop shifting.

Comments

- Given the positions of the extrema, there exists a formula for the optimum δ. However we don't know the optimum δ nor the exact positions of the extrema.
- Thus we need to iterate. Assume the positions of the extrema, calculate δ, move the extrema, recalculate δ, until δ stops changing.
- The algorithm generally converges in about 12 iterations.

Pole-Zero Diagram



Example

- Design a low pass filter with the
- following specifications
  Fs = 10 kHz
  Fc = 1 kHz (-3dB)
  A = 60dB at 1.5 kHz
- From harris formula,
  N >10000x60/(500x22) = 55

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