Observations on Windows

- Windows with low sidelobe levels have large transition bandwidth's.
- Transition bandwidth is inversely proportional to N for a given window.
- Indeed, the ratio of transition width over sidelobe level in dB for fixed length N is virtually a constant.
window	**width**	**sidelobe** (dB)
boxcar	1.0	13
Hann	2.0	31
Blkhar3	3.0	66
Blkhar4	4.0	92

Estimating Filter Length

- We can use the following empirical formula due to harris to estimate the required filter length

$$\Delta f \approx \frac{f_s}{N} k(A) \text{ where } k(A) \approx \frac{A(dB)}{22}$$

$$N \approx \frac{f_s}{\Delta f} \frac{A(dB)}{22}$$

where N is the filter length, A is the out of band attenuation in dB, and f_s is the sampling frequency.

Note: There is no mention of the bandwidth of the filter

More Accurate Formula

- Bellanger's Formula for k in two parameters

$$k(IR, OR) = \frac{-20\log(IR) - 20\log(OR) - 20}{30}$$

where IR is the in band ripple in dB

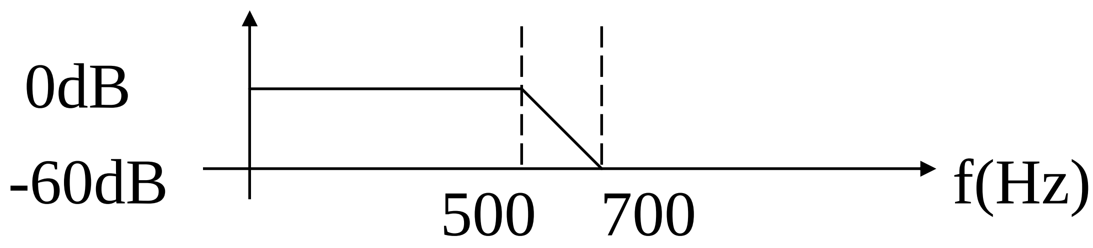
OR is the out of band ripple in dB

harris formula is generally OK since we have to iterate anyway

Example

- Assume 10 kHz sampling frequency
- We want a low pass filter with cutoff frequency of 500 Hz, a transition band of 200 Hz, and an out of band attenuation of 60 dB.
- From harris formula,

$$N \approx \frac{10000}{200} \bullet \frac{60}{22} \approx 136$$



Linear Phase

- FIR filters are generally designed to have a linear phase characteristic.
- This means that all frequency components experience the same time delay.
- This is easily achieved by making sure that the filter impulse response is real and even. In this case, the frequency response is always real – or Hermitian (conjugate symmetric) if we allow for a change of time origin.
- Arbitrary phase responses can be easily obtained by simple cross-multiplication in the freq domain.

Filter Design Procedure

- Determine filter length, N, from transition bandwidth and out of band attenuation.
- Determine required window function, W, from out of band attenuation requirements.
- Create a frequency domain vector , V, of length N or greater with 1's in the pass band and 0's in the stop and transition bands. This is the frequency sampling method, otherwise determine impulse response analytically.
- Inverse DFT V and multiply by W to get the required filter.
- Check filter against specifications by plotting FT of impulse (dft with zero padding).

Worked Example

- Fs = 10 kHz, Fc = 500 Hz, Ft = 200 Hz, A = 60dB as before
- N = 136

$$L = \frac{f_c}{f_s} N = \frac{500}{10000} \bullet 136 = 6.9 \text{ say } 7$$

wdesgn1
zoom

- Choose 3-term Blackman harris (66 dB) as W to meet sidelobe requirements.
- In V, set samples 1 to 8 (7 frequency bins) and 130 to 136 to 1; otherwise 0

Worked Example

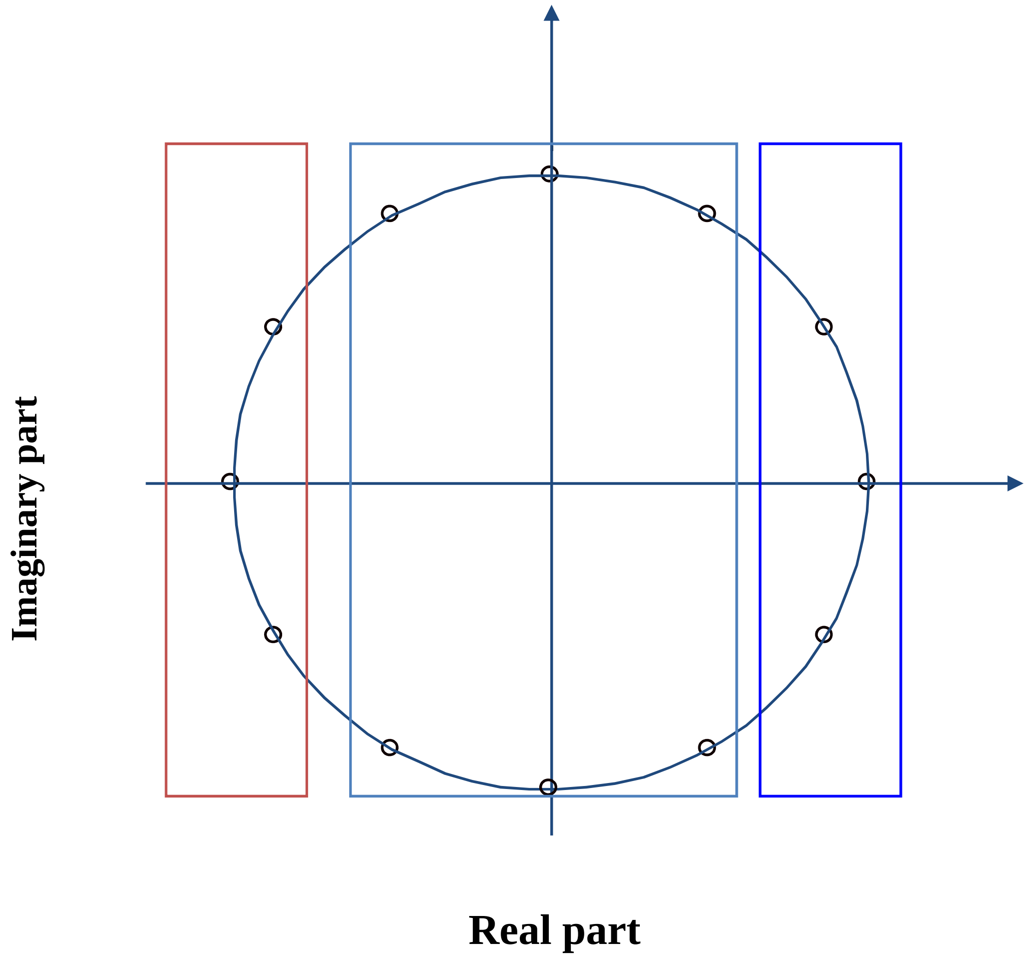
wdesgn2
zoom

- After the design we find that the filter does not satisfy the specifications
- The passband response is 2.4dB down at 500Hz and the stopband rejection is 50dB rather than the design goal of 60 dB.
- In this case we could try to increase the filter length to increase the rolloff in the transition band. Try N=160. This is a good choice because L=8 exactly so the passband requirement should be satisfied exactly.
- In V, set samples 1 to 9 and 153 to 160 to 1; otherwise 0

Worked Example

- After modification, the passband response is 3dB down at 500Hz, and the stopband response is 80dB down at 700Hz, which is 20dB more than we need.
- It would be nice to avoid this clumsy method of guessing a window function and seeing if it meets the specifications.
- Kaiser windows are a parameterised family of functions which can meet specified pass and stopband ripple requirements.
- Watch out for “one off “errors and “DFT symmetry” errors. See Matlab code.

Window Lengths: N Even



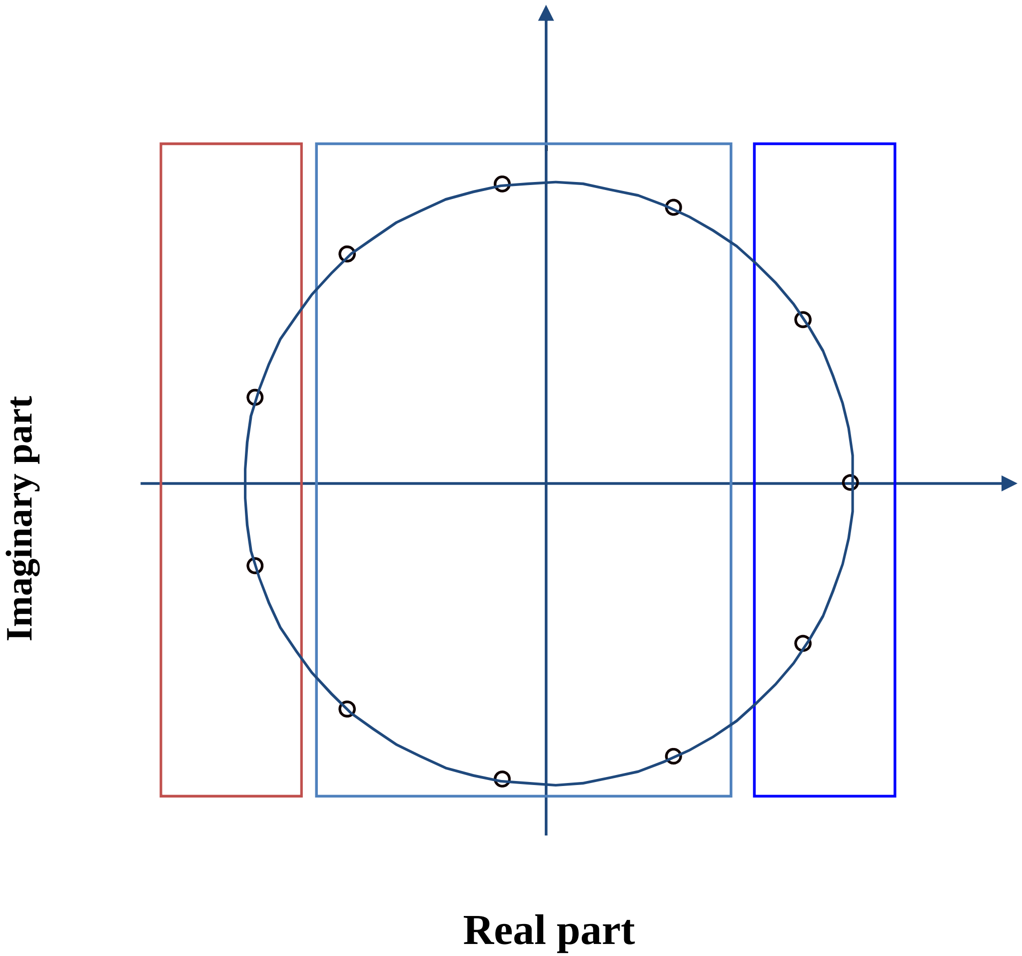
N=12 even

LP is odd

BP is even

HP is odd

Window Lengths: N Odd



N=11 odd

LP is odd

BP is even

HP is even

Kaiser Window Design

$$w(n) = \frac{I_0(\beta\{1-[2n/(N-1)]^2\}^{1/2})}{I_0(\beta)}, |n| \le (N-1)/2$$

$$\beta = 0, A \le 21\text{dB}$$

$$\beta = 0.5842(A-21)^{0.4} + 0.07886(A-21),\ 21 < A < 50$$

$$\beta = 0.1102(A-8.7), A \ge 50$$

where A = 20log10(δ) is the required attenuation

Kaiser Window Design

$$\delta = \min(\delta_p, \delta_s)$$

δ_p desired passband ripple

δ_s desired stopband ripple

$$N = \frac{A - 7.95}{14.36(\Delta f / f_s)}$$

Kaiser Example

passband	150-250 Hz
transition width	50Hz
passband ripple	0.1dB
stopband attenuation	60dB
sampling frequency	1kHz

Use the Kaiser window method of design

Solution

kwdesgn

$$20\log(1-\delta_p) = -0.1\,\text{dB},\ \delta_p = 0.0114$$

$$20\log(\delta_s) = -60\,\text{dB},\ \delta_s = 0.001$$

So

$$\delta = \min(\delta_p, \delta_s) = 0.001$$

Now

$$N \geq \frac{A-7.95}{14.36(\Delta f / f_s)} = \frac{60-7.95}{14.36(50/1000)} = 72.49$$

Let

$$N = 73$$

Thus

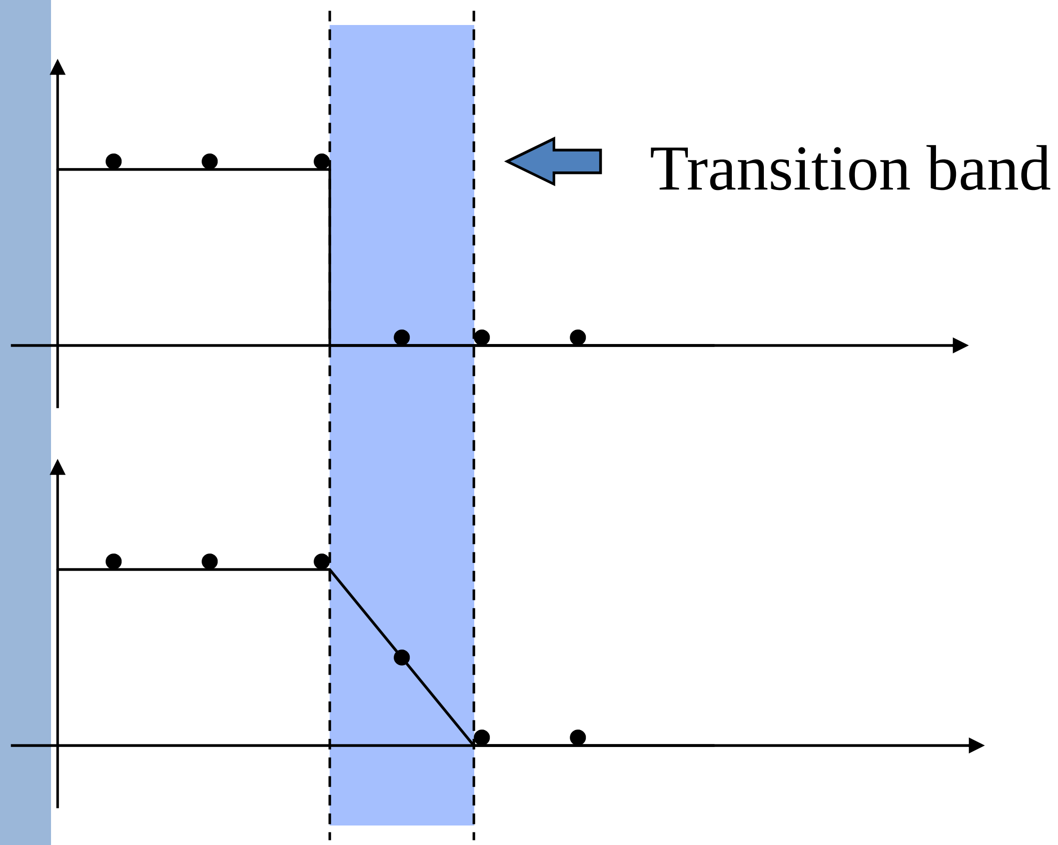
$$\beta = 0.1102(60-8.7) = 5.65$$

Comments on Window Design

- Simple, easy to apply, minimal computation.
- Lack of flexibility. Both the peak passband and stopband ripples are approximately equal, so the designer is often forced to exceed some design specifications at the expense of longer filters.
- Passband and stopband edge frequencies cannot be specified precisely.
- Need to select the right window to meet stopband specifications.
- Errors due to frequency sampling of ideal filter response.

Frequency Sampling

iwdes



Adding some points in the transition band can improve passband ripple and stopband attenuation at the expense of transition bandwidth

This is really just equivalent to windowing the impulse response!

(conv in freq domain)

For one transition choose T between 0.25 and 0.45

Rabiner et al (1970)