Non-Recursive Filters

FIR Filters

All Zero Filters

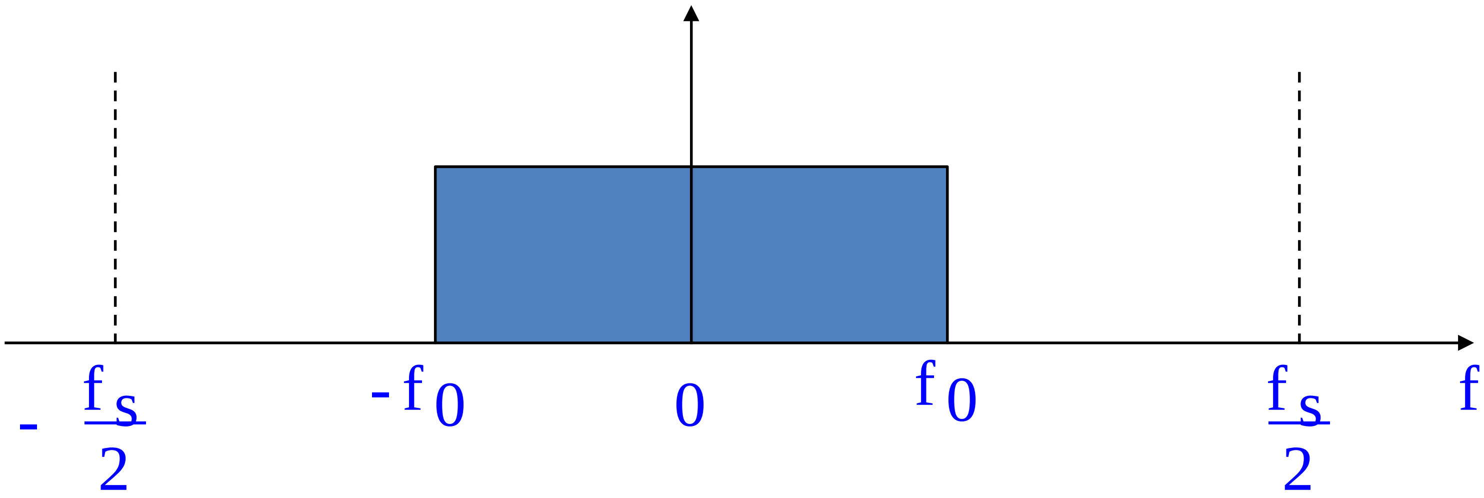
Moving Average Filters

Filter Design

- Let us examine a low pass non-recursive filter with the following characteristics
  - N = 9 samples
  - f_s = 2000 Hz (sample freq)
  - f_0 = 200 Hz (cutoff freq)

Note: assume time is continuous for the moment

Magnitude Response



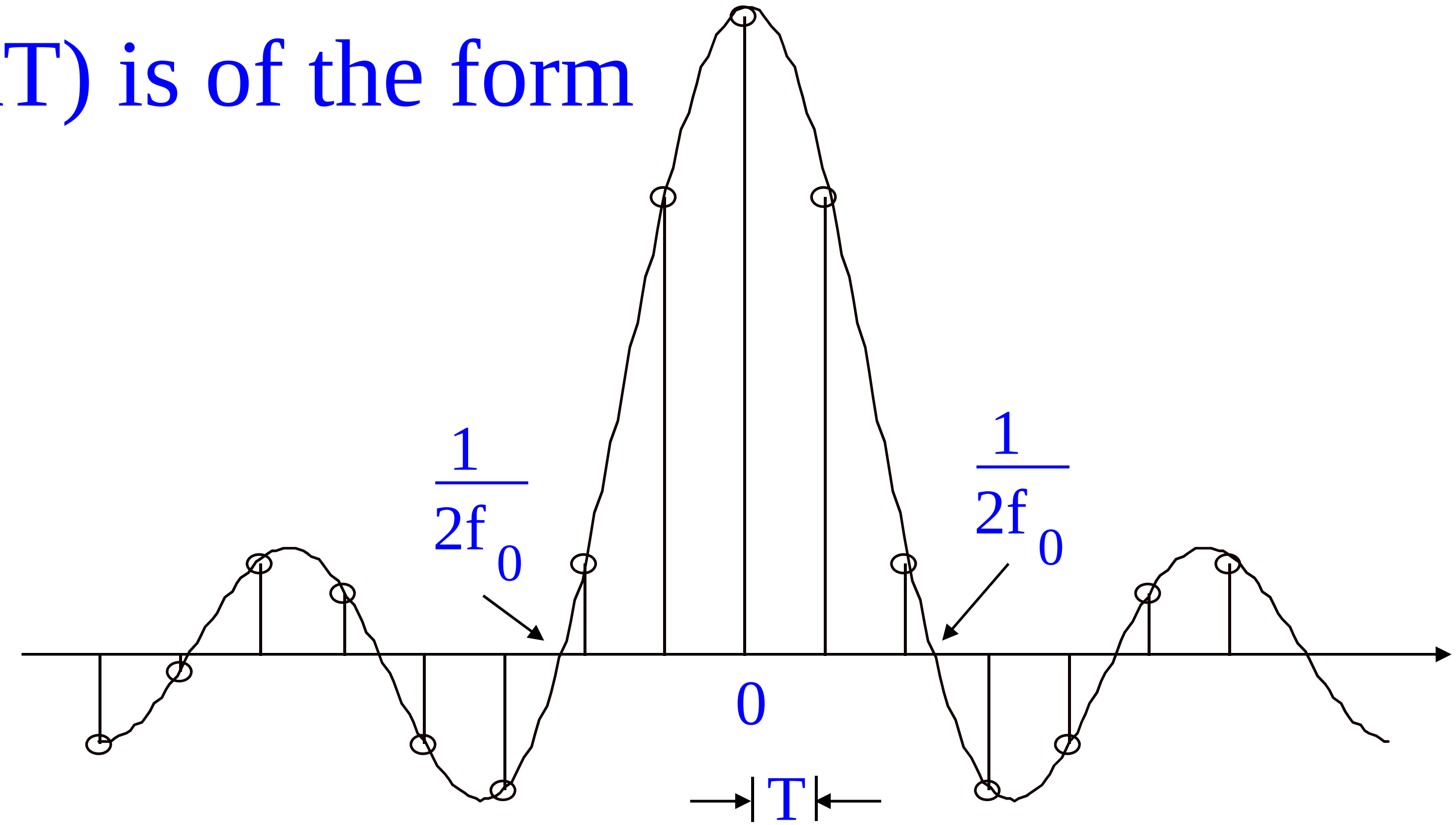
Impulse Response

$$h(kt) = h(t)\Big|_{t=\frac{2\pi}{\omega_s}k} = \frac{\omega_0}{\pi}\frac{\sin \omega_0 t}{\omega_0 t}\Big|_{t=\frac{2\pi}{\omega_s}}$$

$$\frac{\omega_0}{\pi}\frac{\sin \frac{\omega_0}{\omega_s}2\pi k}{\frac{\omega_0}{\omega_s}2\pi k}$$

Impulse Response

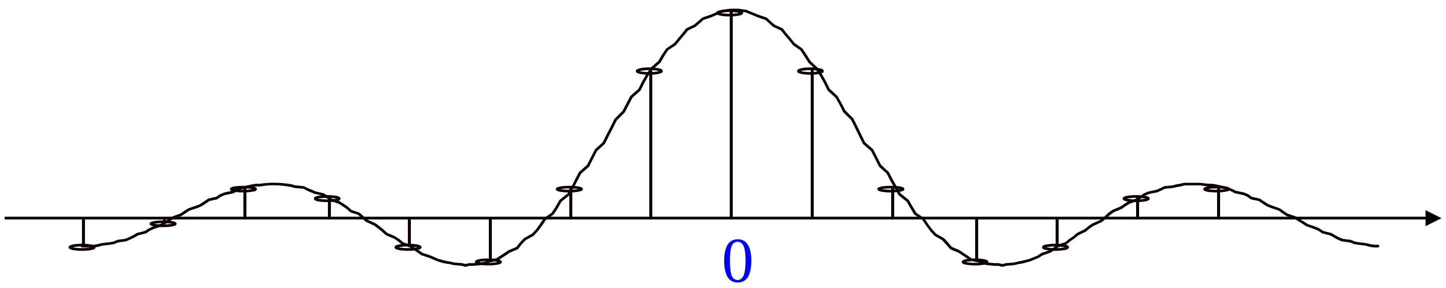
h(kT) is of the form



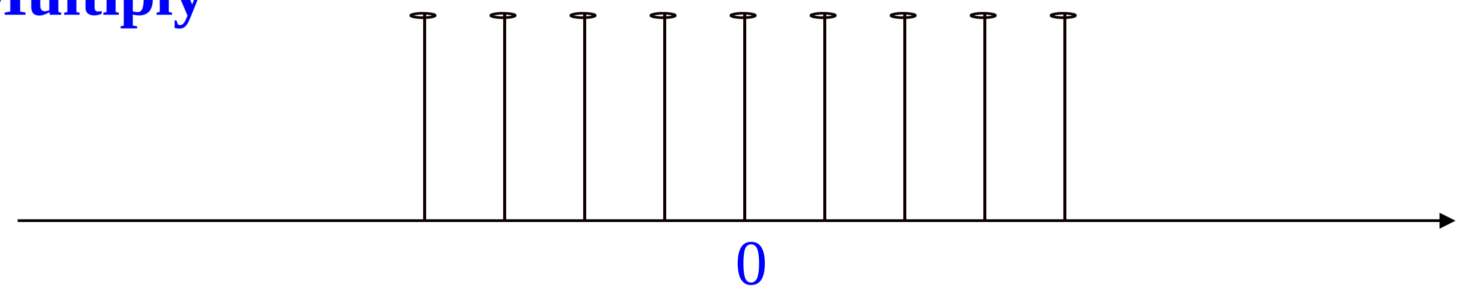
Truncation

- To realise the frequency response we are required to use all of the infinite samples in the impulse response. But we are building a filter with a finite time response! We can do this by simply truncating the above series; we can set the coefficients past the fourth to zero by multiplying by a function which is 1.0 out to the fourth sample, and is zero elsewhere.
- Another word for truncation is **windowing**.

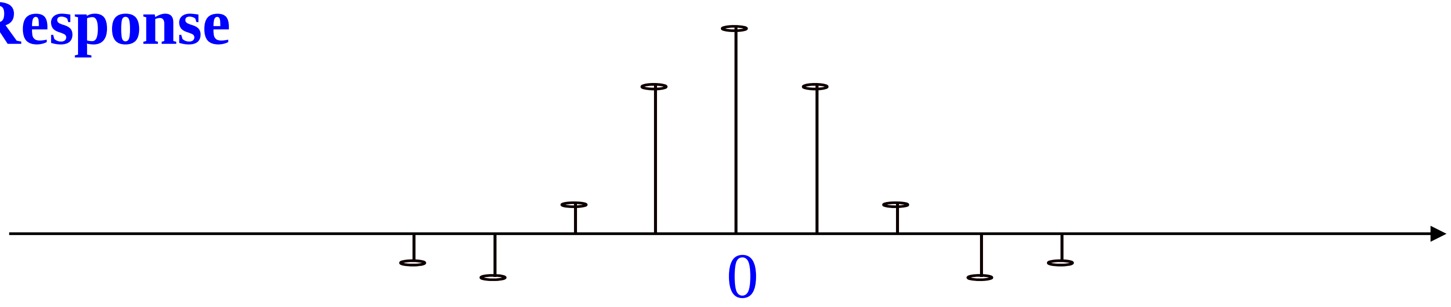
Impulse Response



Multiply

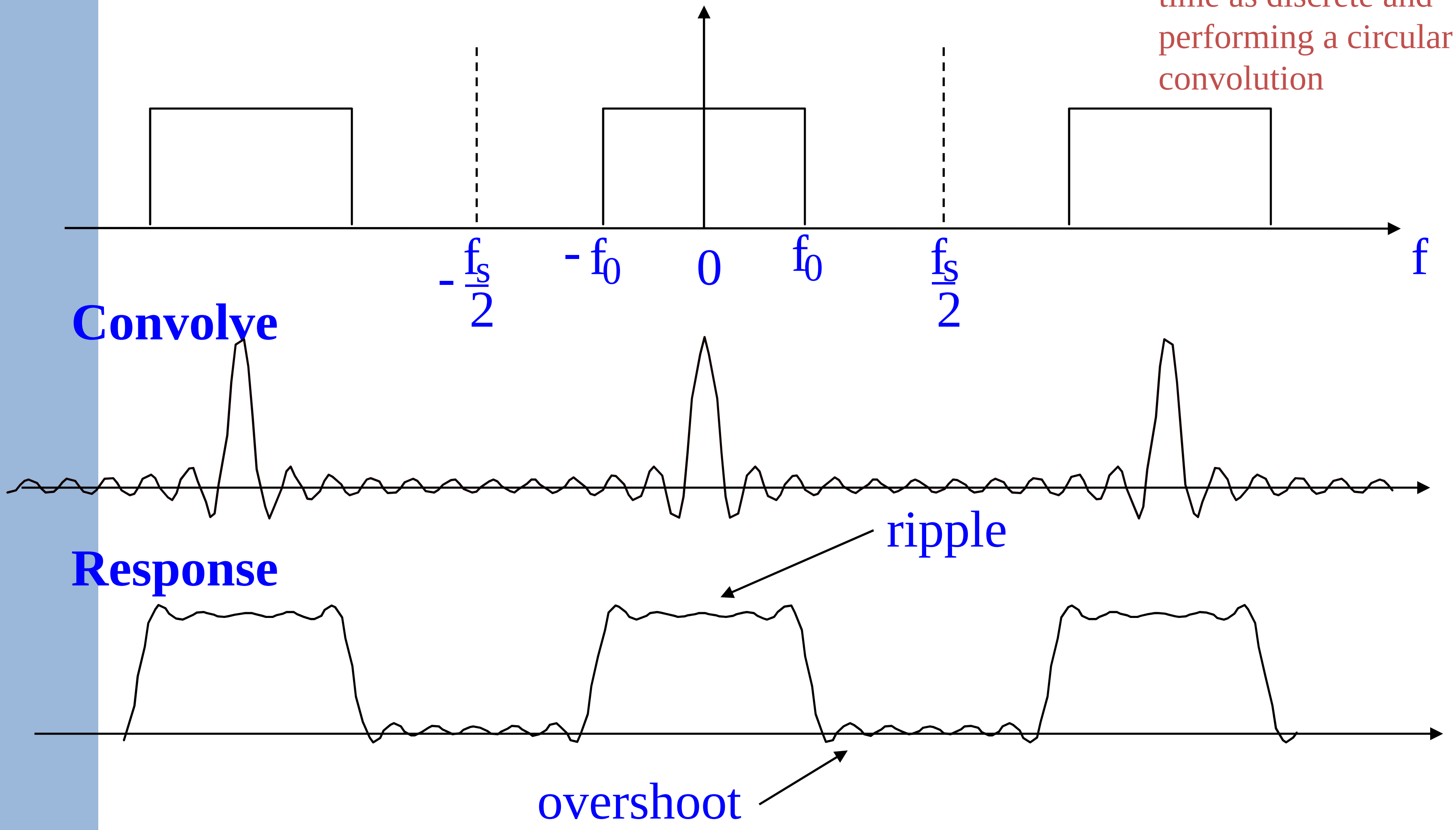


Response



Frequency Response

Note: In this case we are treating time as discrete and performing a circular convolution

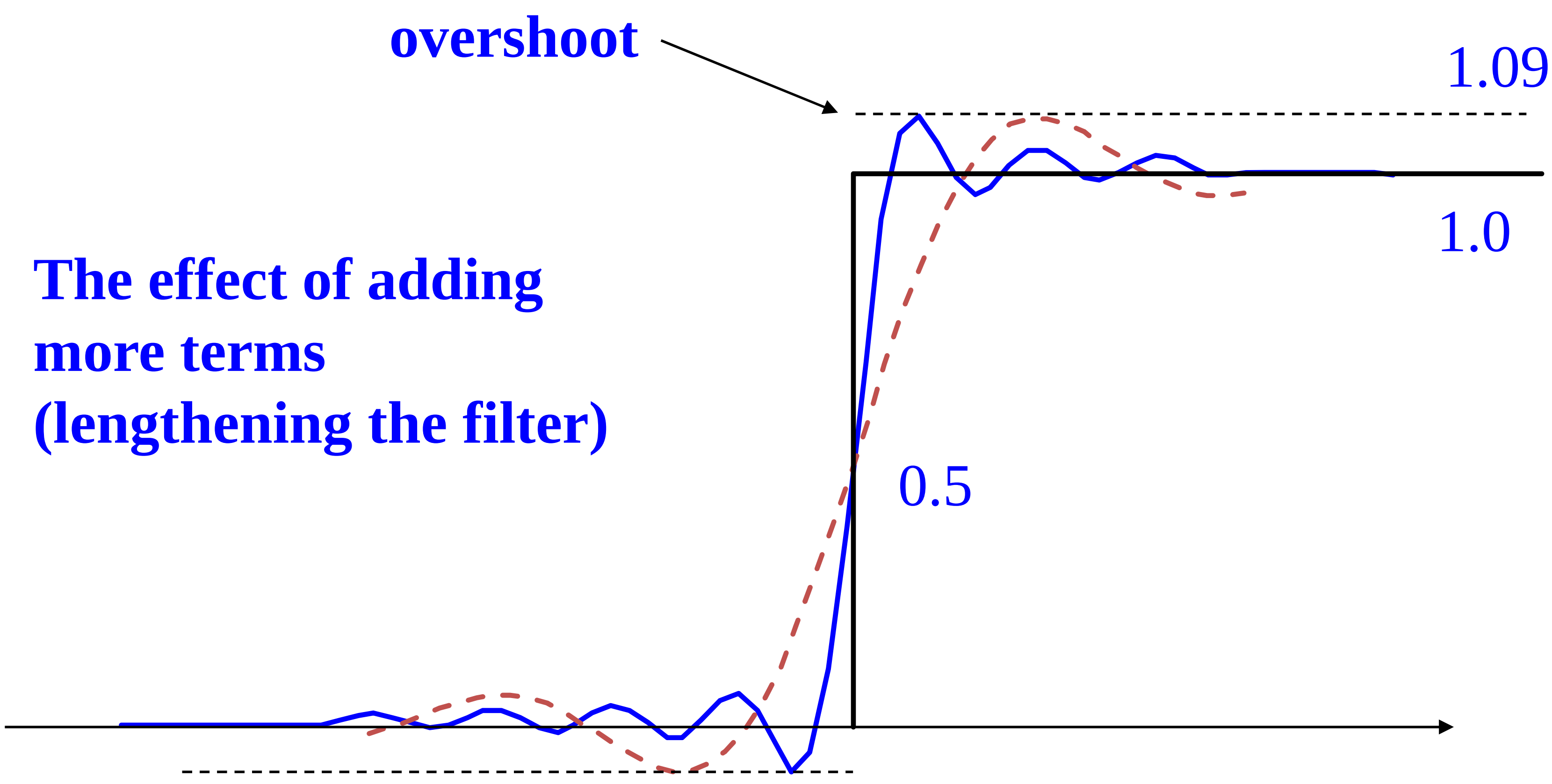


Comments

- The overshoot and ripple in the frequency domain is the price we pay for truncating the impulse response.
- It is the result of selecting the Fourier coefficients to minimise the mean squared error between the function and its Fourier series approximation.
- We, of course, recognise it as **Gibb's** phenomenon.
- The poor behaviour in the neighbourhood of the discontinuity is the result of convolving with a "bumpy" function.

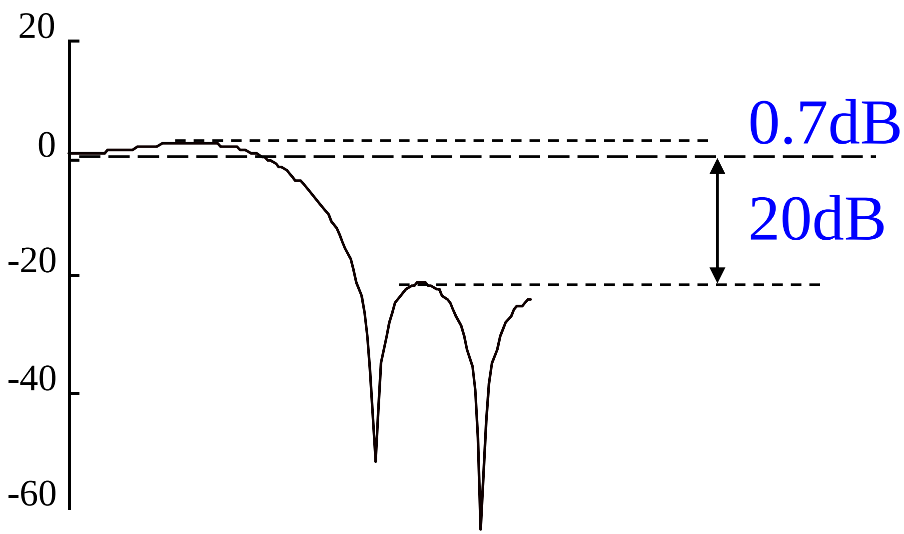
- Gibb's phenomena is **not** due to the fact that only a finite number of terms have been employed in the reconstruction!
- The magnitude of the peak ripple is invariant to the number of terms in the series.
- Adding more terms simply moves the ripple towards the discontinuity; it does not change its structure.

Gibb's Effect



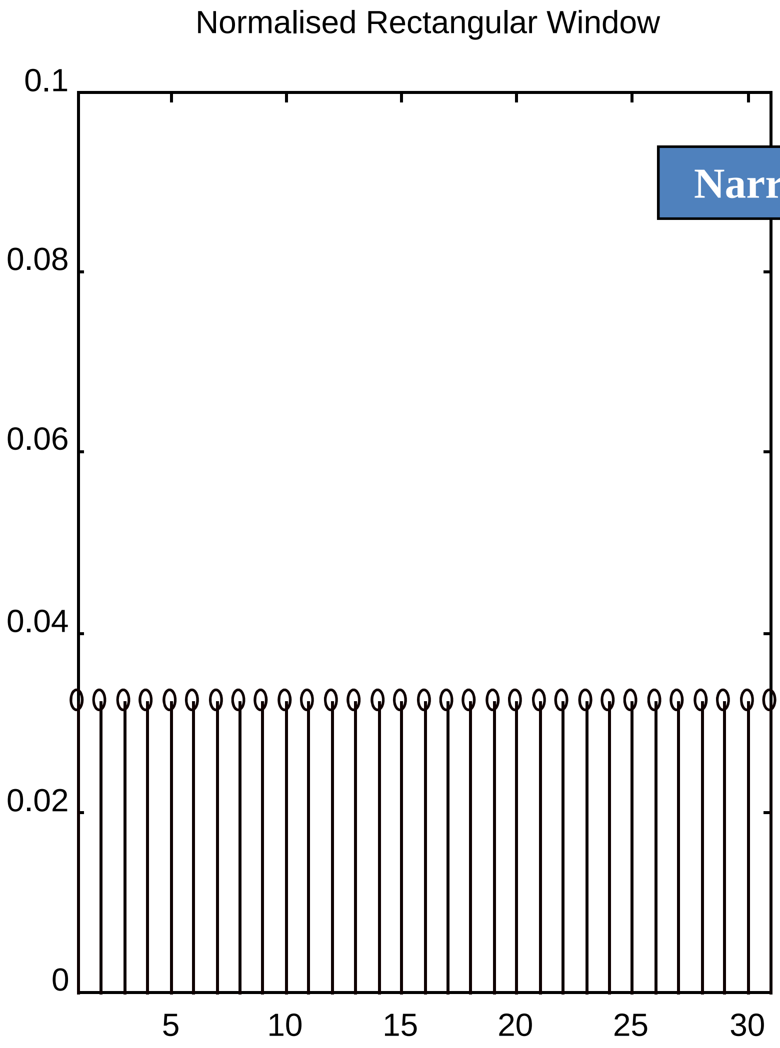
Comments

- Note that ripple in the neighbourhood of 10% overshoot in the passband represents only 0.7 dB, but that 10% ripple in the stop band represents 20 dB.

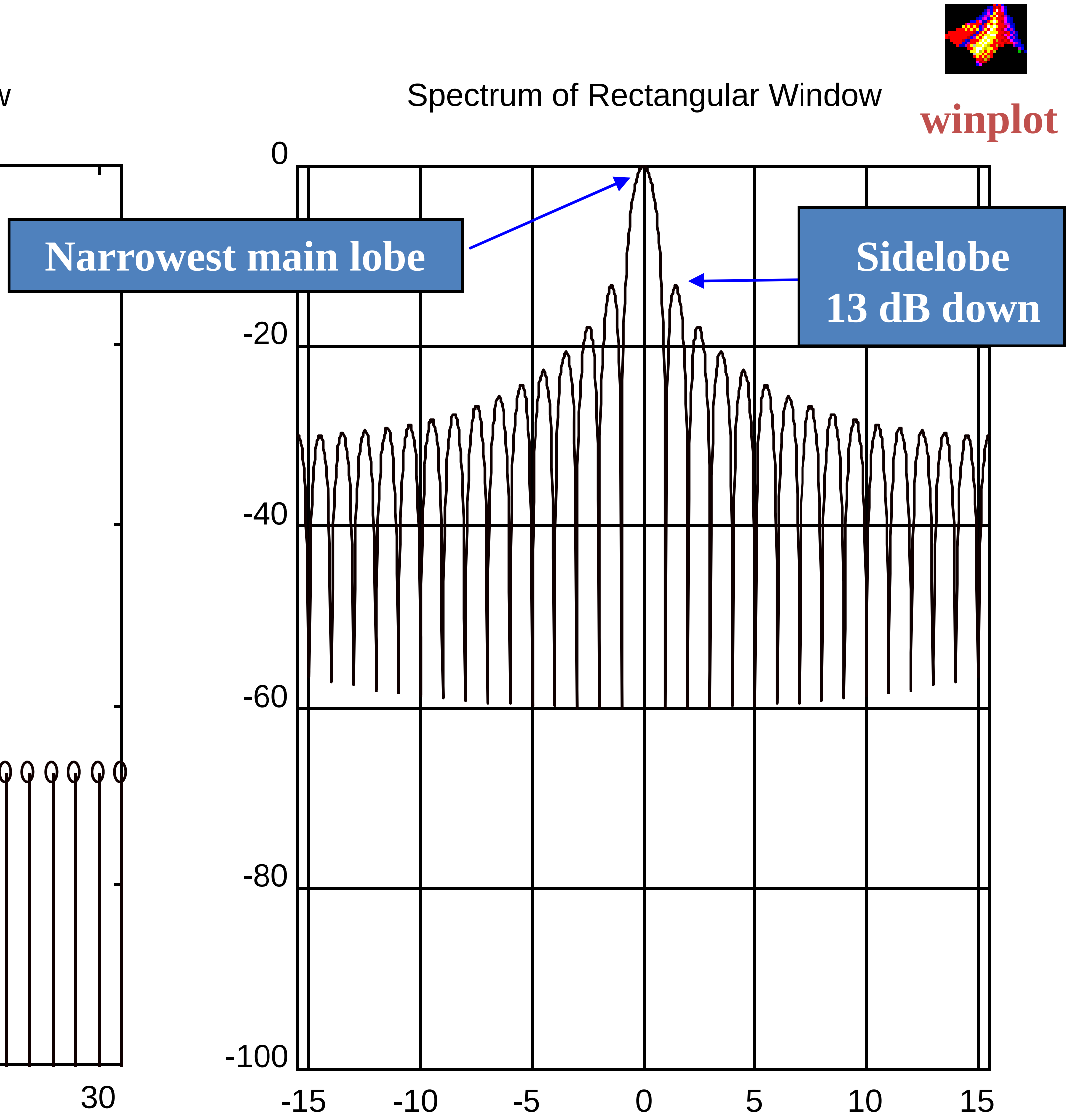


- We would probably find this filter inadequate to meet filtering requirements in the stop band.
- How do we improve performance?
- We have commented that the poor ripple characteristics are due to the bumpy function used in the convolution.
- To reduce the ripple, we need a truncation function whose Fourier Transform is not as bumpy (i.e., smoother)
- The bumpy behaviour of the transform is related to the discontinuity of the truncation function.

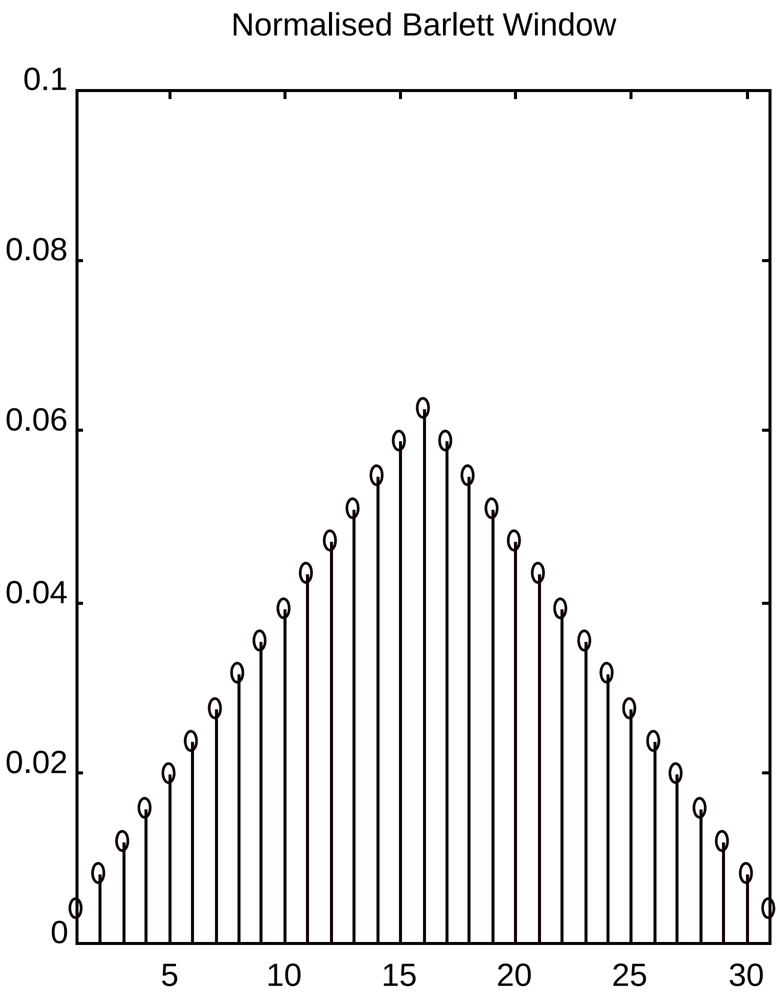
- The study of smooth truncation functions classically referred to as **windows** is a very important, but not well-understood segment of DSP.
- Misconceptions abound!
- What follows is an assortment of common window functions followed by a series of important observations.



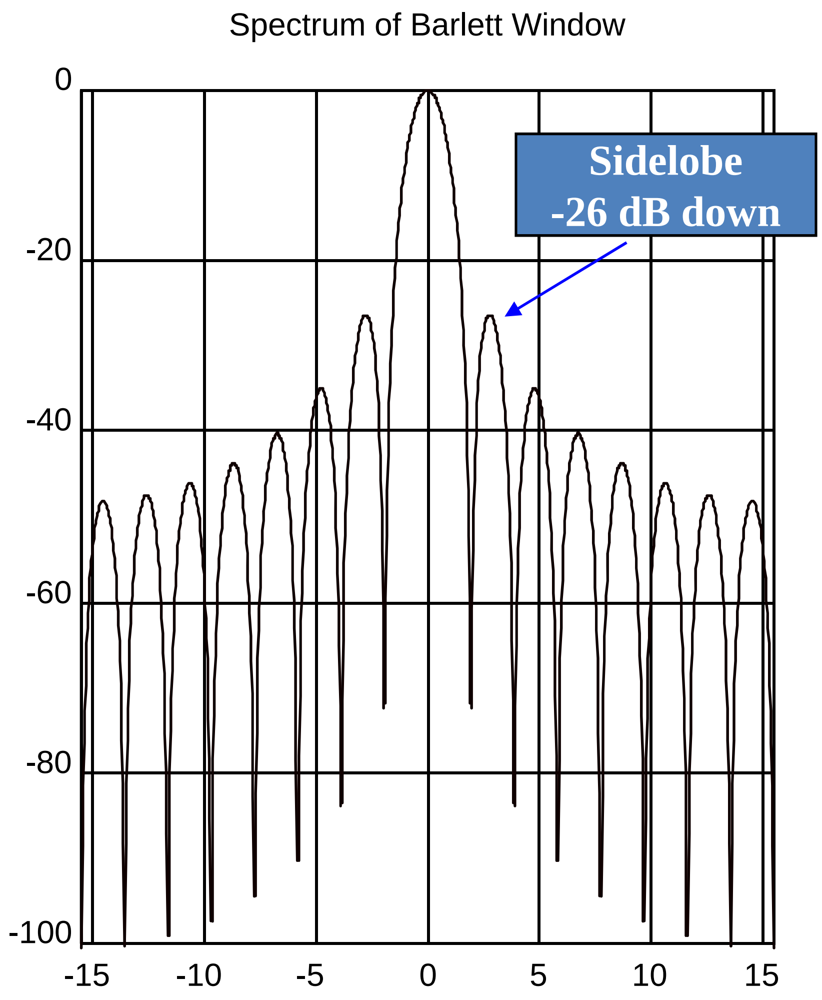
Normalised Rectangular Window
0.1
0.08
0.06
0.04
0.02
0
5
10
15
20
25
30



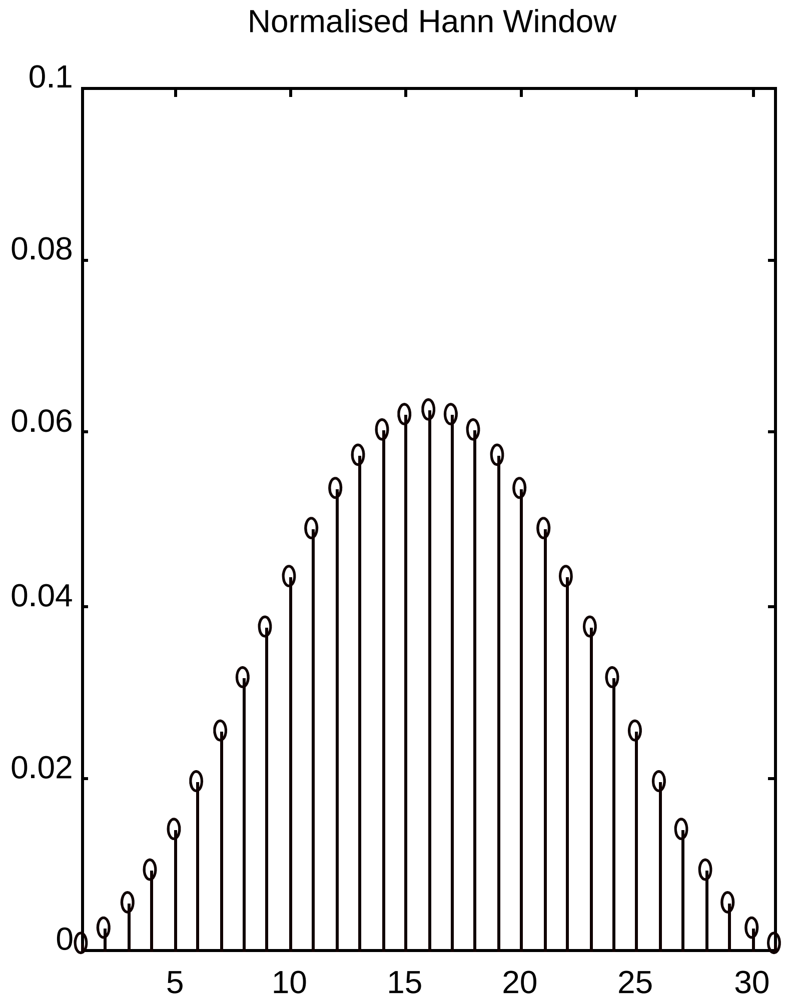
Spectrum of Rectangular Window
0
-20
-40
-60
-80
-100
-15
-10
-5
0
5
10
15
Narrowest main lobe
Sidelobe
13 dB down
winplot

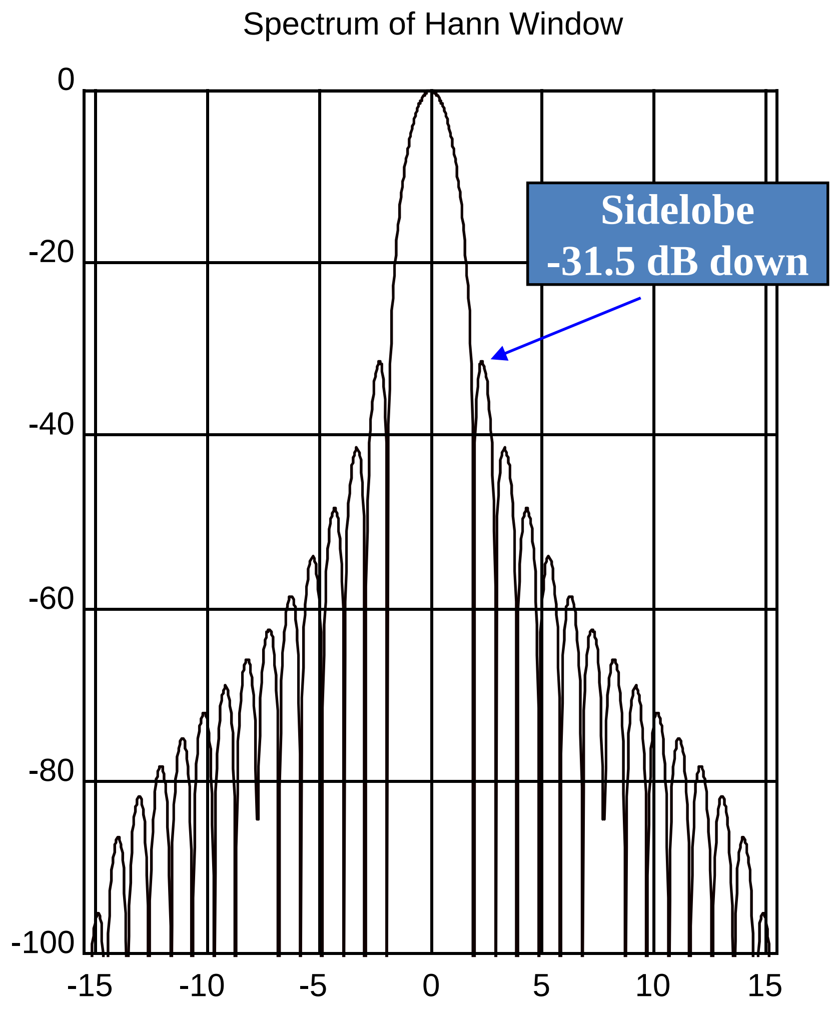


Normalised Barlett Window
0.1
0.08
0.06
0.04
0.02
0
5
10
15
20
25
30

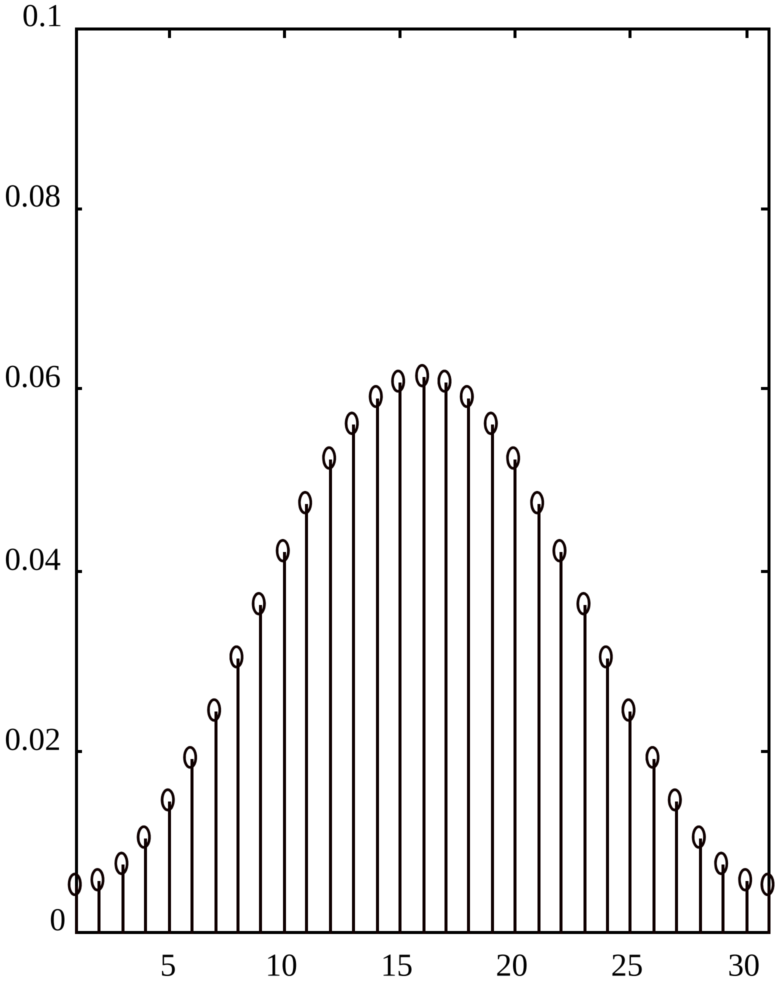


Spectrum of Barlett Window
Sidelobe
-26 dB down
0
-20
-40
-60
-80
-100
-15
-10
-5
0
5
10
15

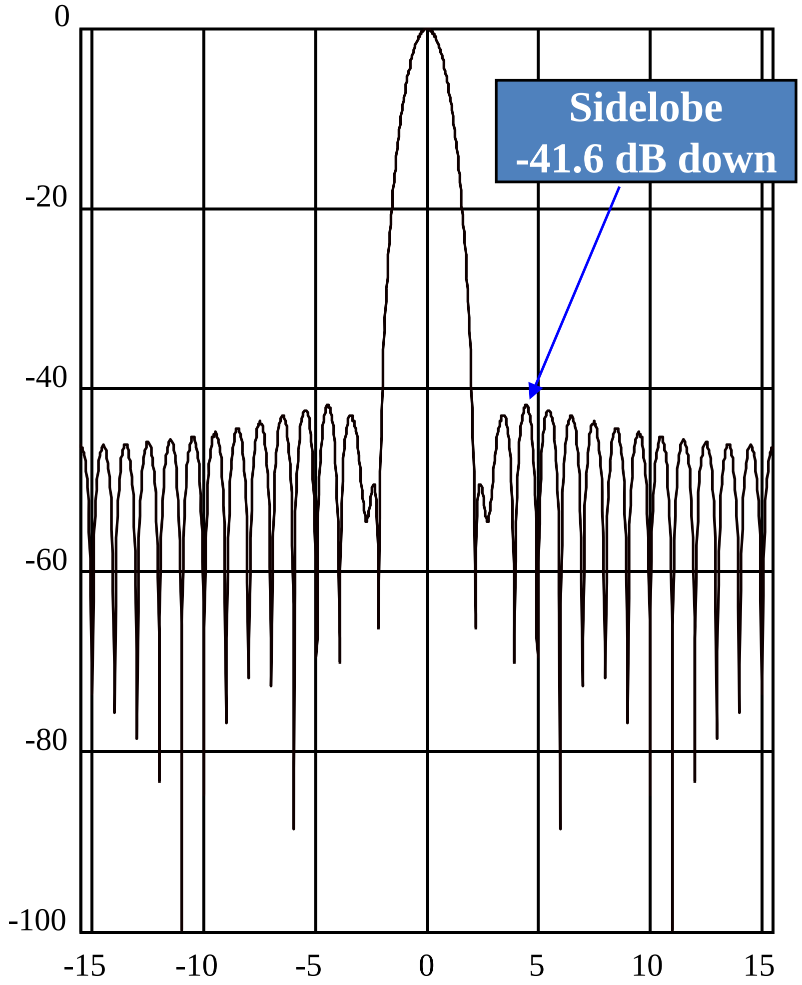

Normalised Hann Window
0.1
0.08
0.06
0.04
0.02
0
5
10
15
20
25
30

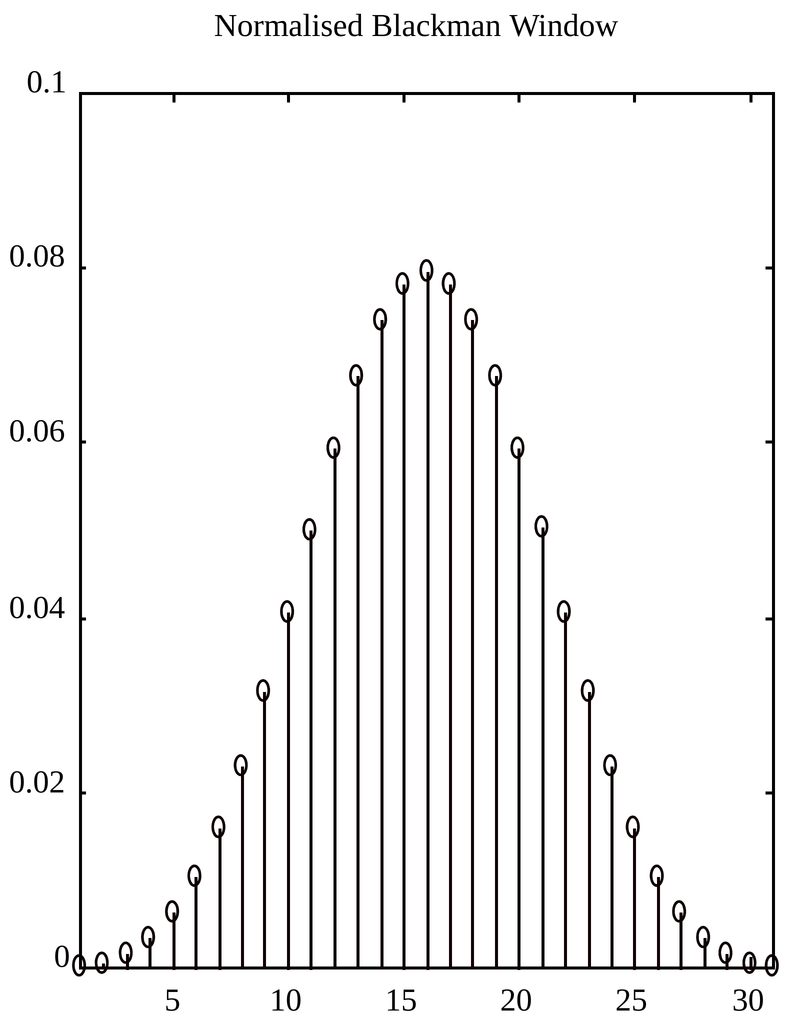

Spectrum of Hann Window
0
-20
-40
-60
-80
-100
-15
-10
-5
0
5
10
15
Sidelobe
-31.5 dB down

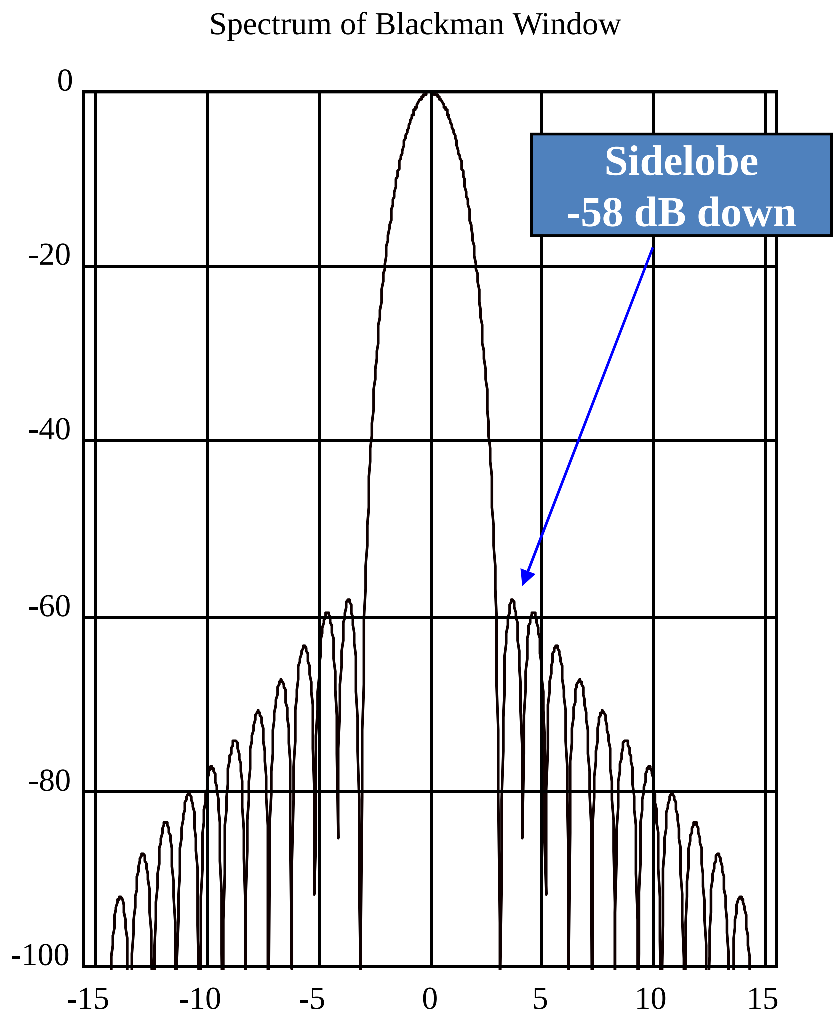
Normalised Hamming Window

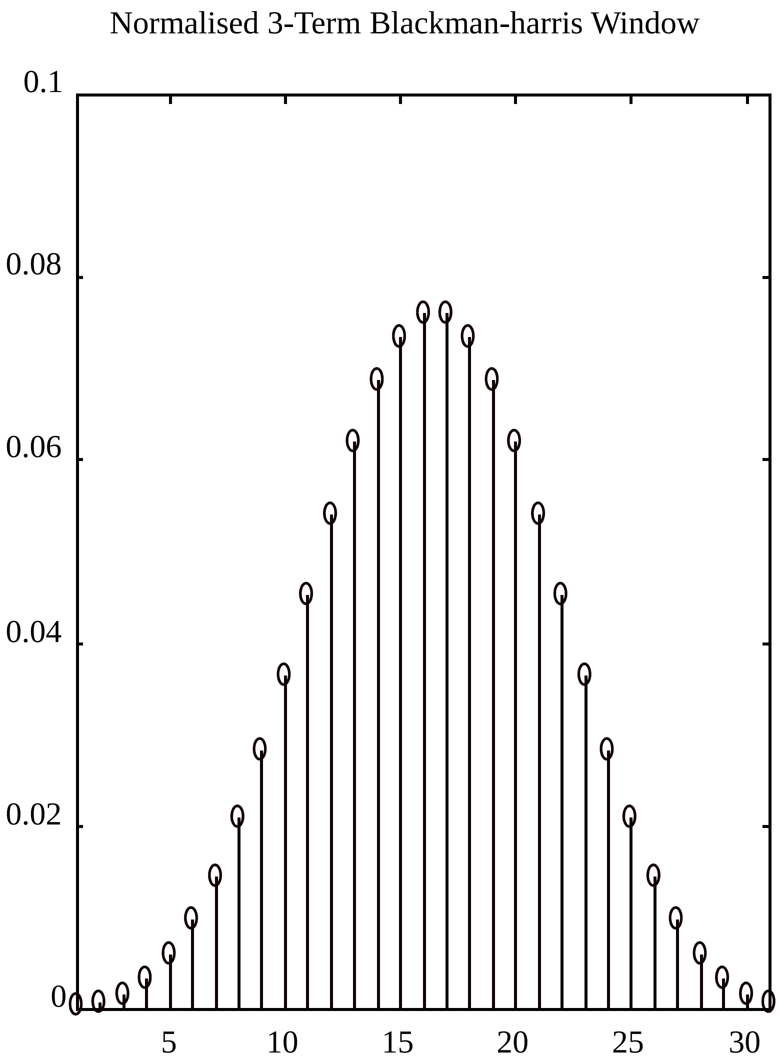


Spectrum of Hamming Window

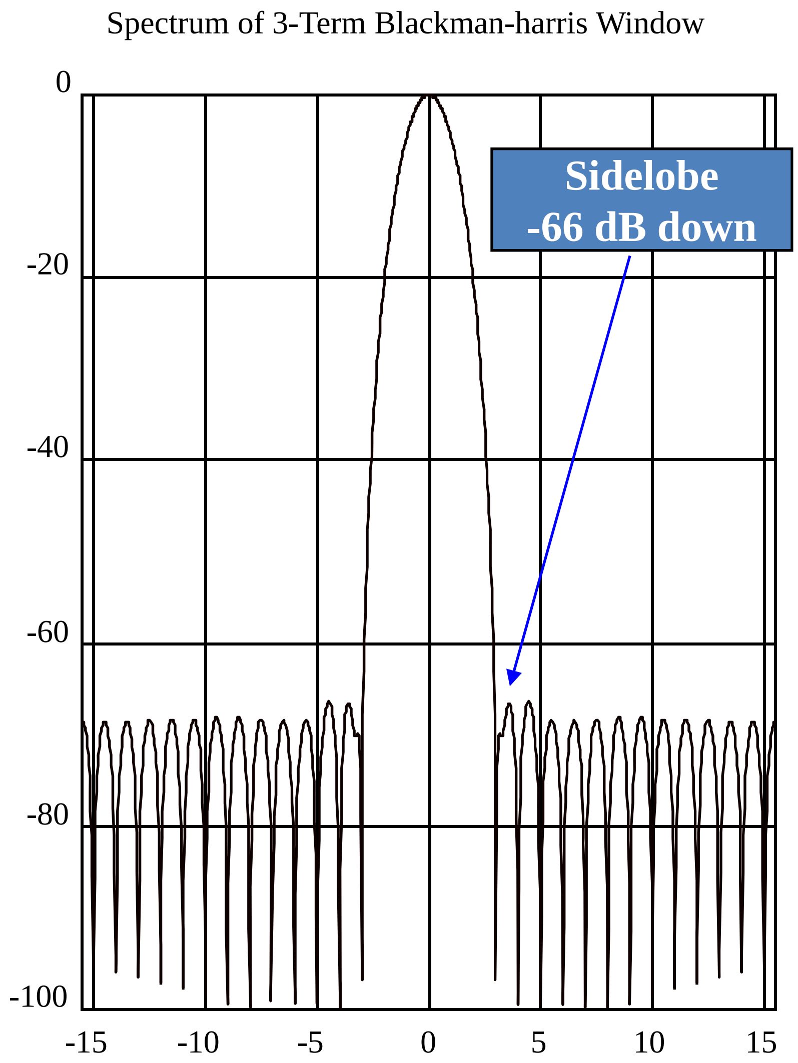



Normalised Blackman Window
0.1
0.08
0.06
0.04
0.02
0
5
10
15
20
25
30

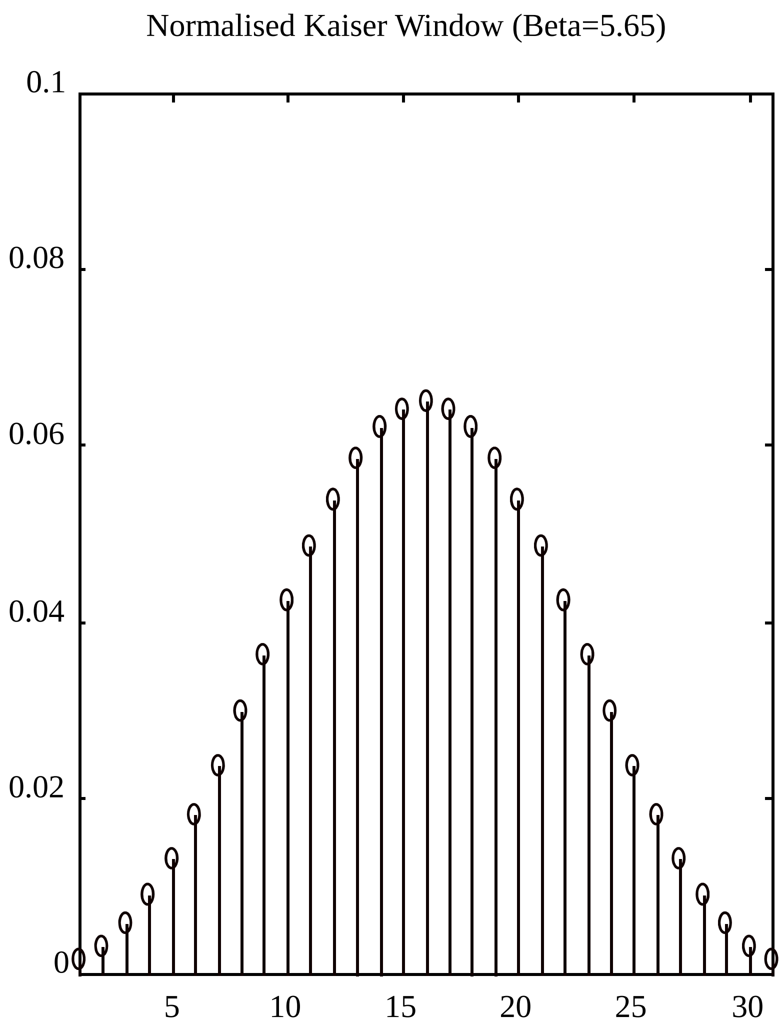

Spectrum of Blackman Window
0
-20
-40
-60
-80
-100
-15
-10
-5
0
5
10
15
Sidelobe
-58 dB down

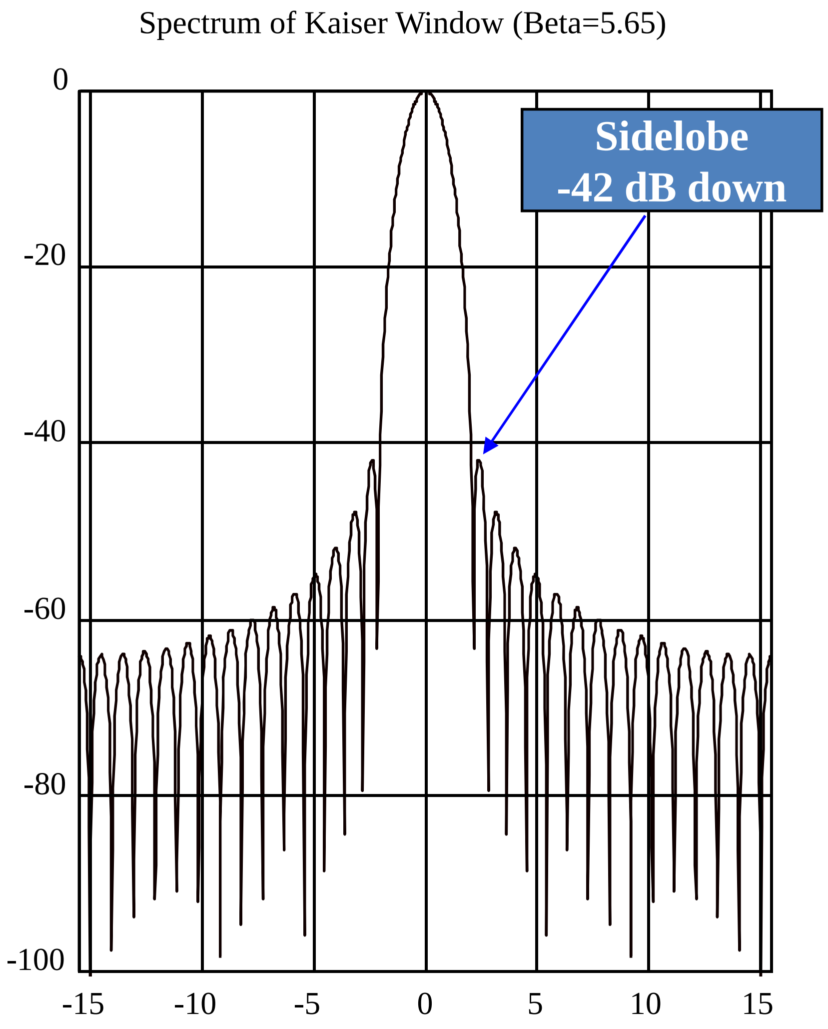


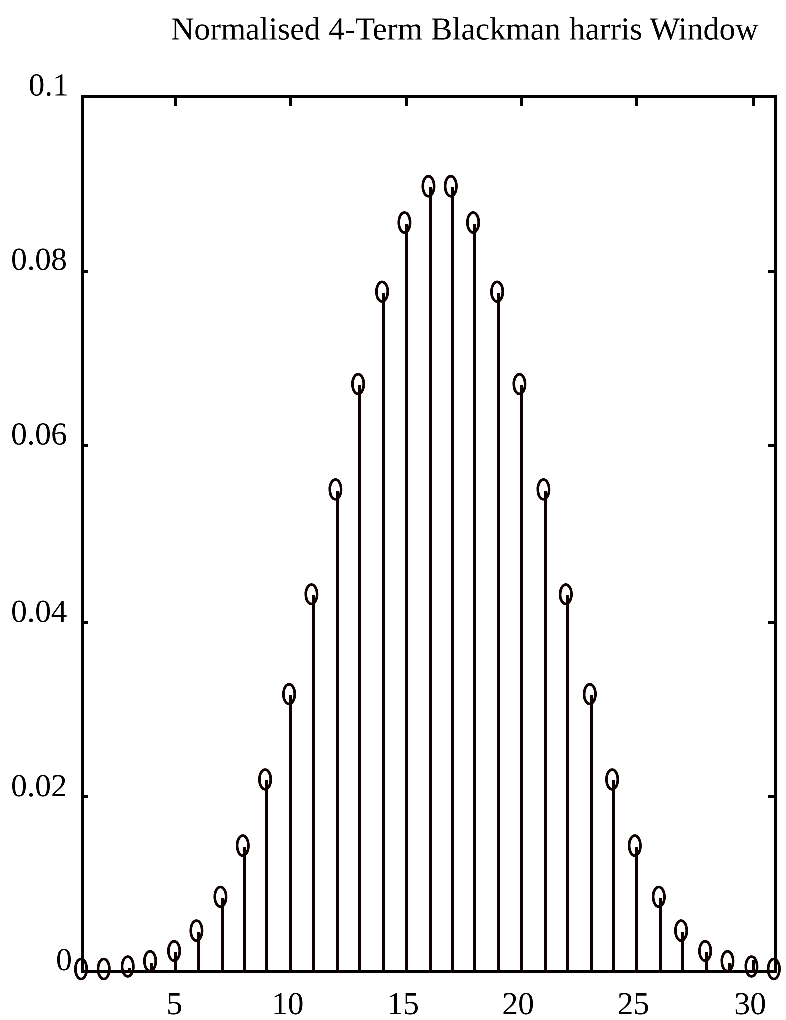
Normalised 3-Term Blackman-harris Window
0.1
0.08
0.06
0.04
0.02
0
5
10
15
20
25
30

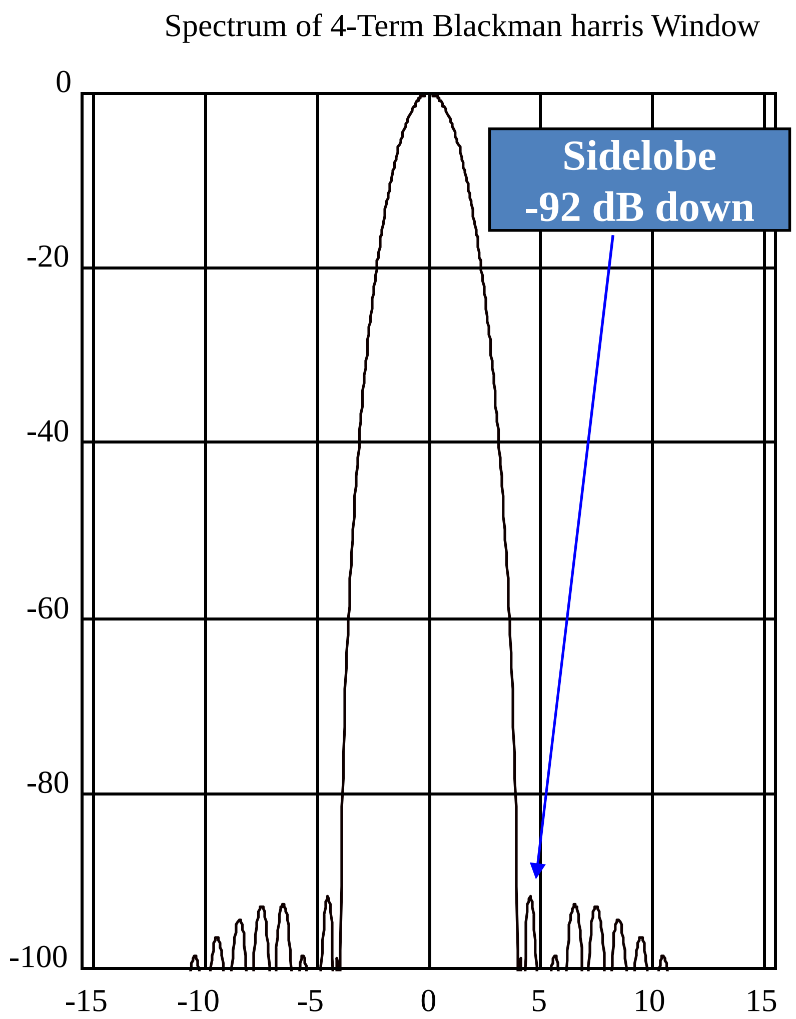


Spectrum of 3-Term Blackman-harris Window
Sidelobe
-66 dB down
0
-20
-40
-60
-80
-100
-15
-10
-5
0
5
10
15


Normalised Kaiser Window (Beta=5.65)
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30

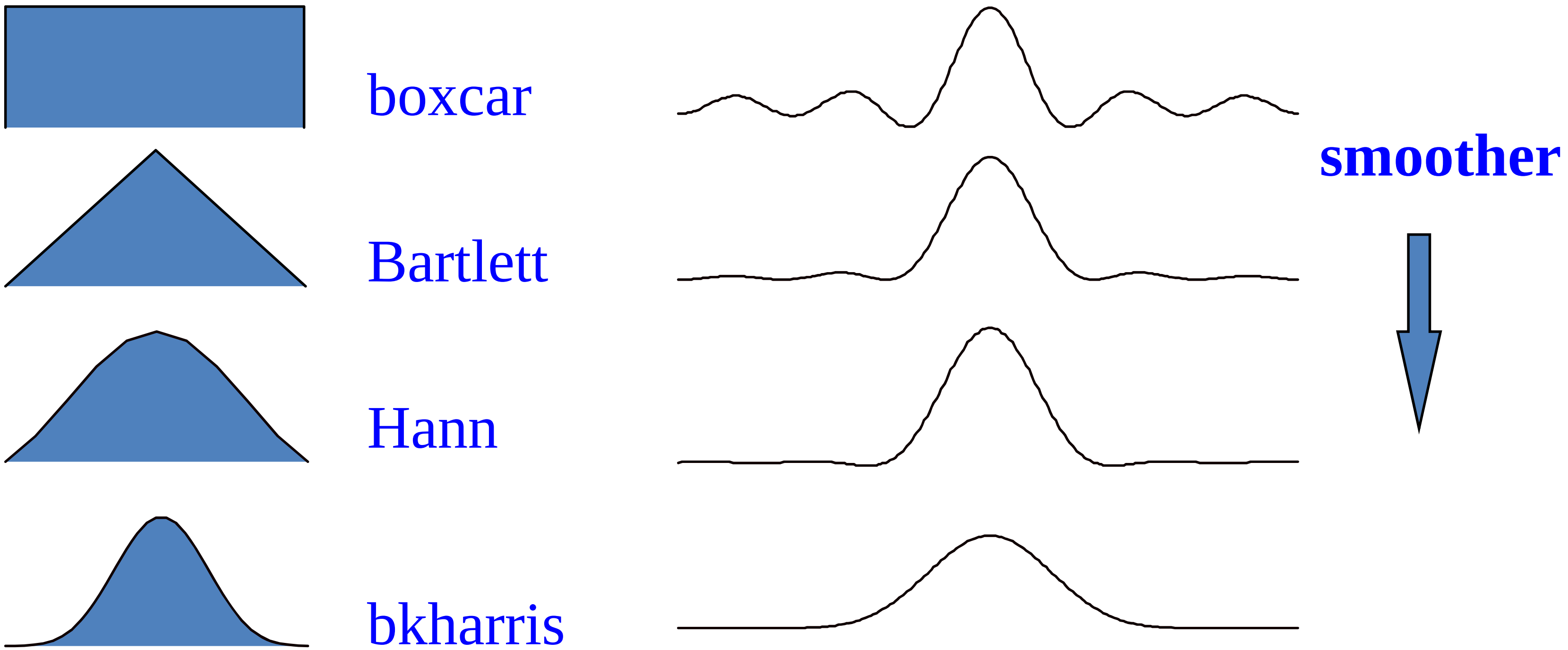

Spectrum of Kaiser Window (Beta=5.65)
Sidelobe
-42 dB down
0
-20
-40
-60
-80
-100
-15
-10
-5
0
5
10
15


Normalised 4-Term Blackman harris Window
0.1
0.08
0.06
0.04
0.02
0
5
10
15
20
25
30


Spectrum of 4-Term Blackman harris Window
0
-20
-40
-60
-80
-100
-15
-10
-5
0
5
10
15
Sidelobe
-92 dB down

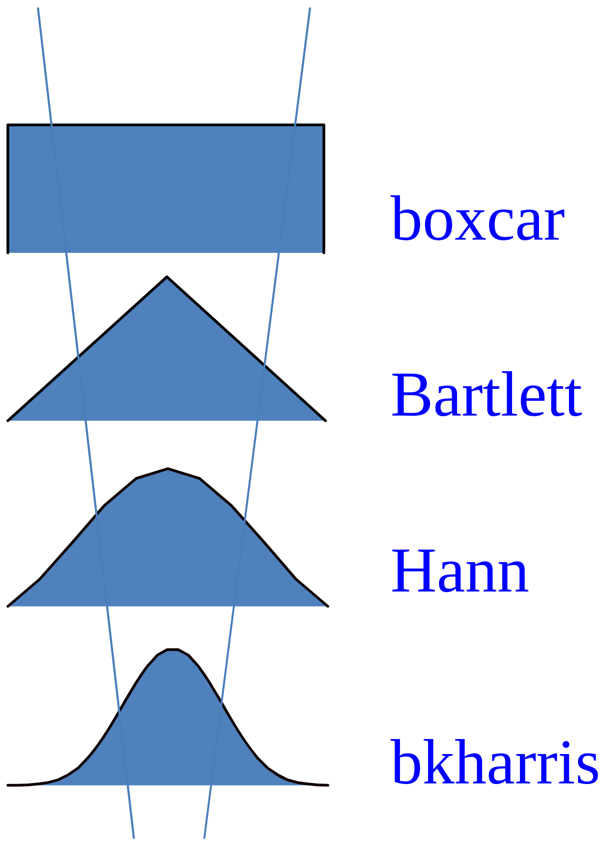
Windows

- Windows with “smoother” behaviour in the time-domain are “smoother” in the frequency domain.



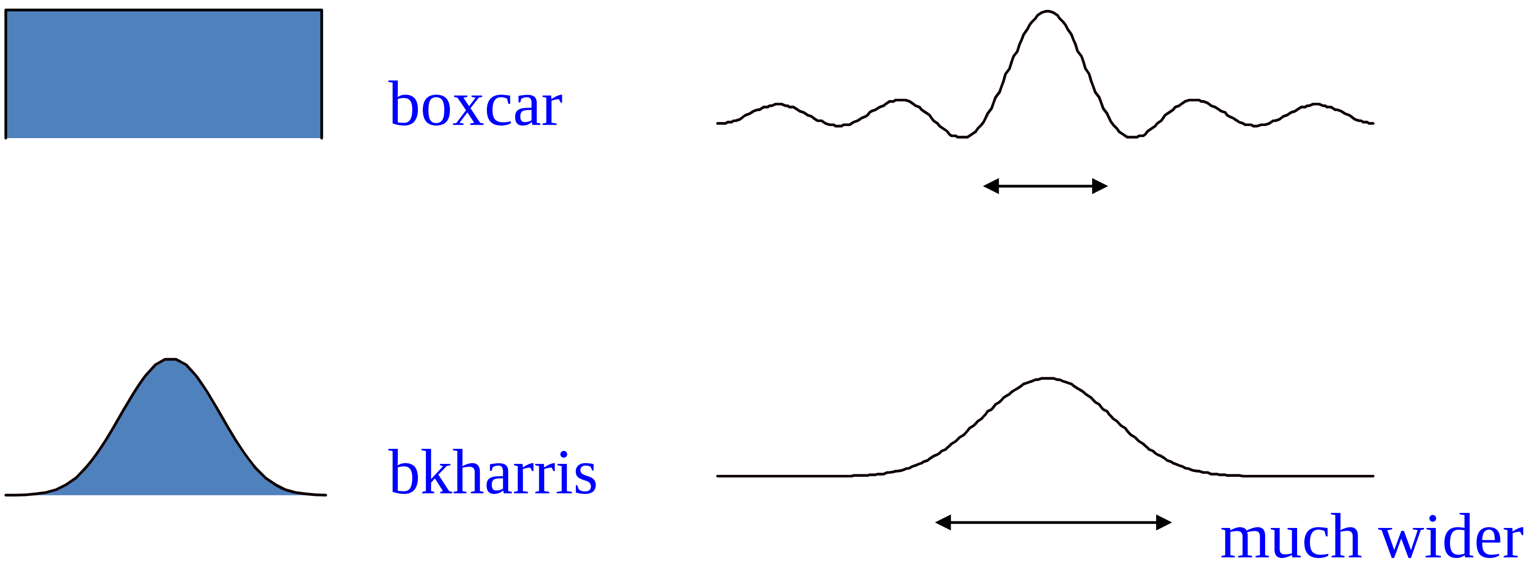
Windows

- Windows that are "smoother" in the time-domain tend to have **narrower** time duration (e.g., smaller rms time-widths).



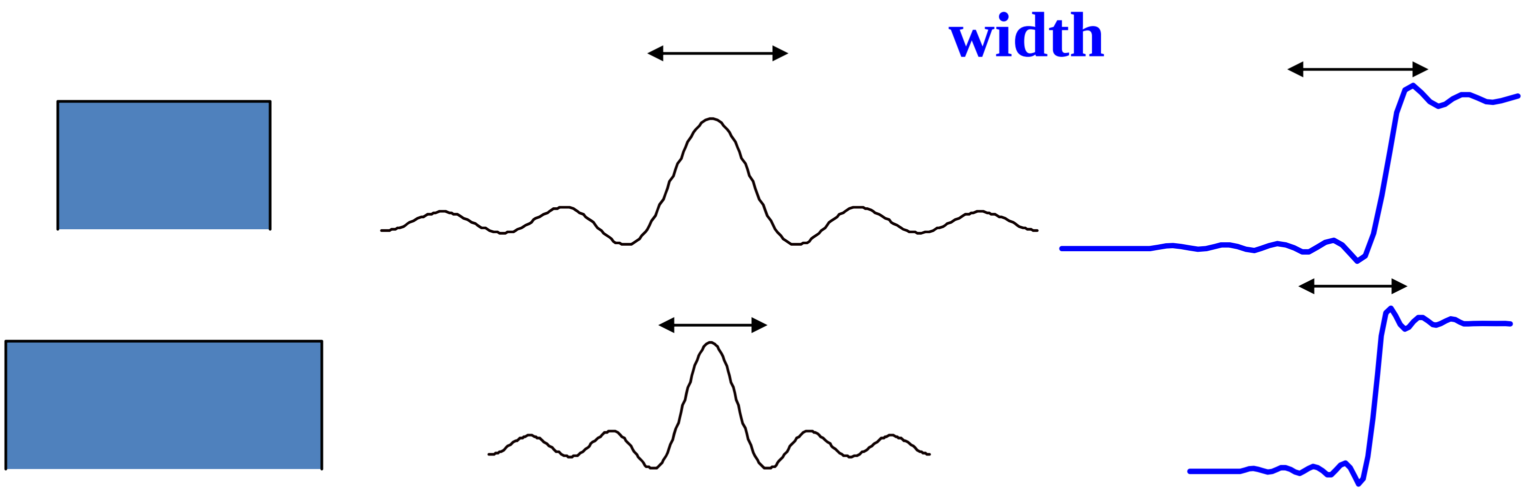
Windows

- Windows that are "smoother" in the time domain tend to have **wider** bandwidths (e.g., greater rms bandwidths)



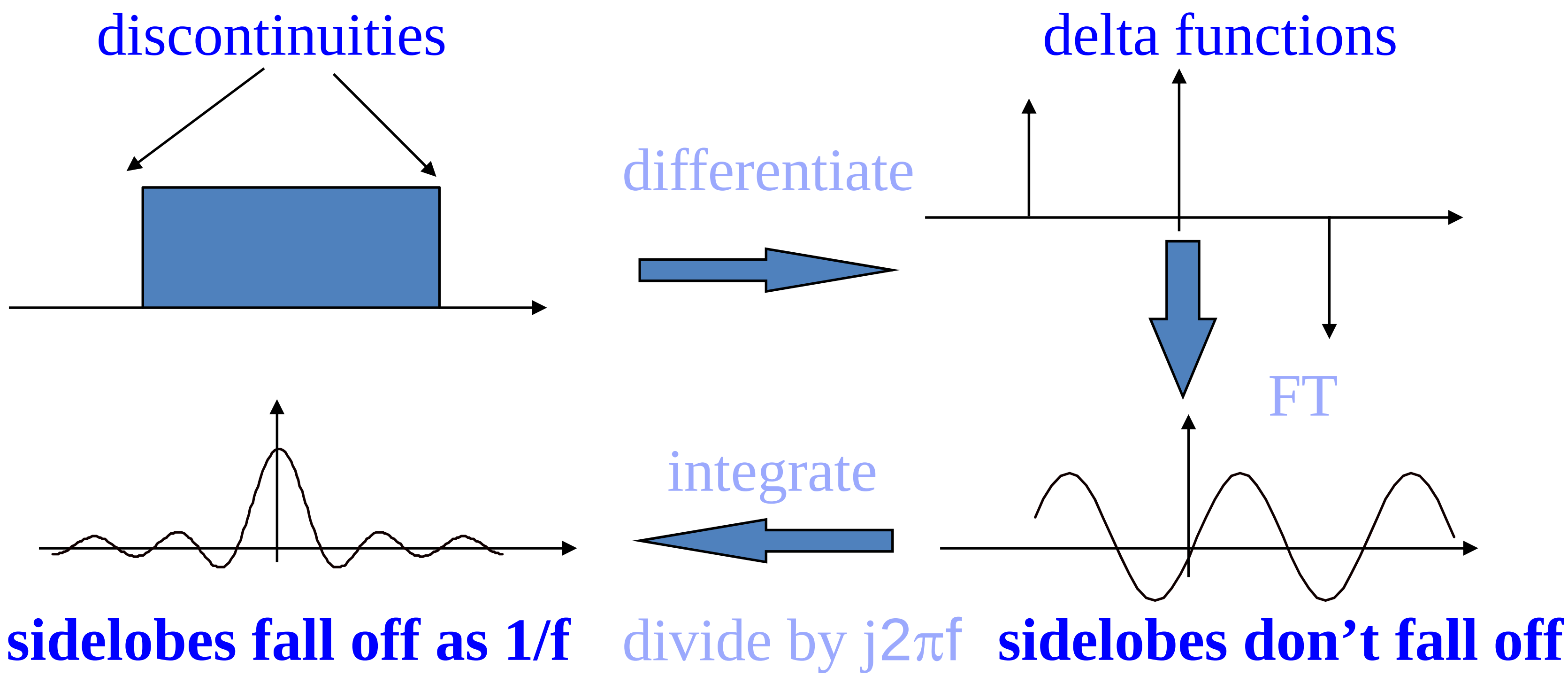
Windows

- When convolving with a **wider** function, the resultant function will have wider transitions (smaller slope).



Smoother Windows

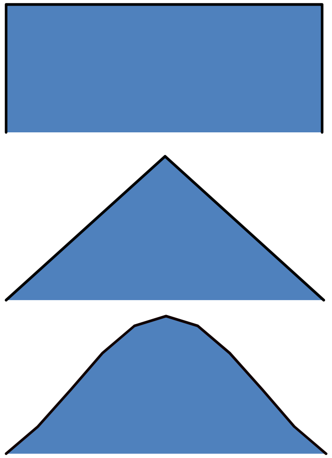
- One mathematical interpretation of "smoother" is continuous derivatives.



Smoother Windows

- So a delta functions in the nth derivative implies that sidelobes fall off as $1/f^n$.

sidelobes fall off as



	Sidelobes fall off as
boxcar	$\frac{1}{f}$
Bartlett	$\frac{1}{f^2}$
	$\frac{1}{f^3}$

windows get narrower

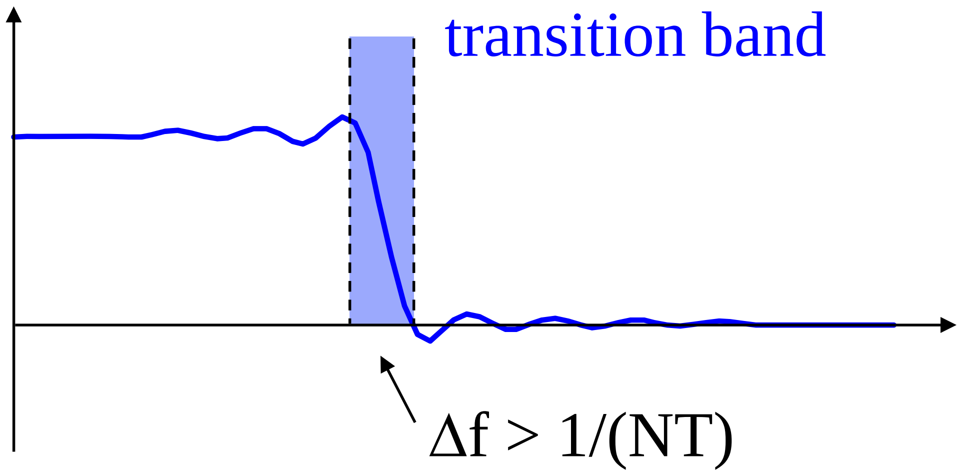
Need continuous derivatives

Sidelobes

- Don't want constant sidelobes.
- Why?
- When we downsample all the noise adds up.
- Always use windows which have some sidelobe fall-off . (About 3dB per octave is sufficient)

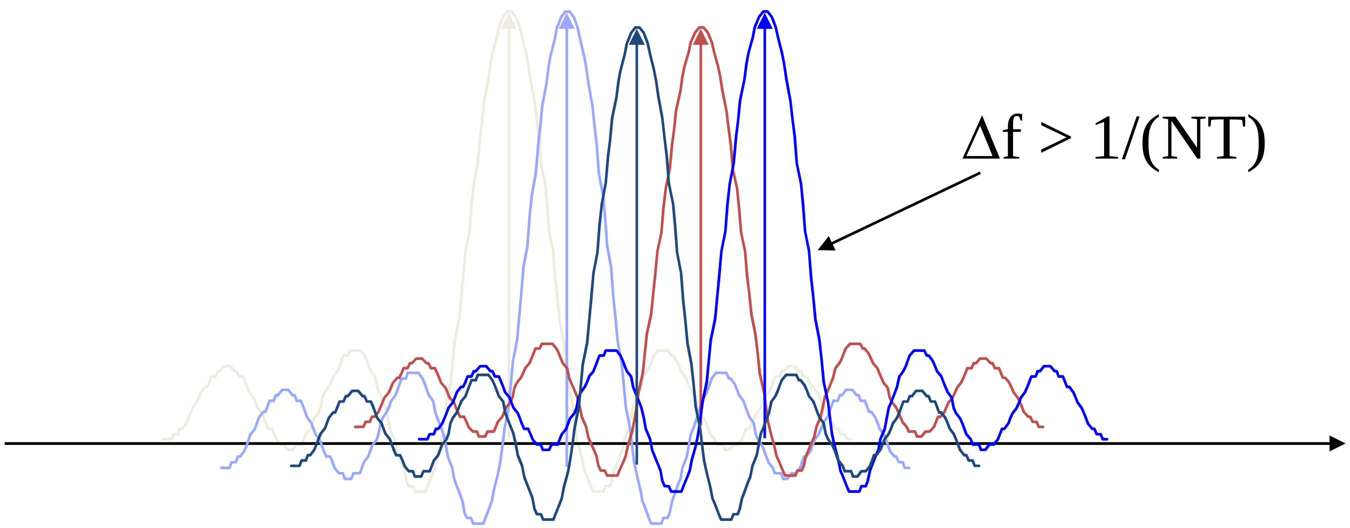
Transition Width

- The boxcar window gives the fastest rolloff in the transition band but has large sidelobes in the stop band.
- We trade rolloff for sidelobe suppression by using more sophisticated window functions.



Another View

- If we use a boxcar function, the rolloff in the transition zone is determined by the final sinc function and this is the fastest possible rolloff that can be achieved.



Optimal Windows

- Dolph-Chebyshev
  - Minimum main lobe width for lowest sidelobe level
- Kaiser-Bessel
  - Window is a good approximation to finite prolate spheroidal functions; minimum time bandwidth product
  - Family of windows with different sidelobe levels
- Blackman-harris
  - Minimum sidelobe for a fixed number of spectral lines