Non- Recursive Filters

FIR Filters
All Zero Filters
Moving Average Filters
Filter Design

- Let us examine a low pass non-recursive filter with the following characteristics
  - $N = 9$ samples
  - $f_s = 2000$ Hz (sample freq)
  - $f_0 = 200$ Hz (cutoff freq)

Note: assume time is continuous for the moment.
Impulse Response

\[ h(kt) = h(t) \left| \begin{array}{c} t = \frac{2\pi k}{\omega_s} \\ \omega_0 \frac{\sin \omega_0 t}{\pi \omega_0 t} \end{array} \right| \left| \begin{array}{c} t = \frac{2\pi}{\omega_s} \\ \omega_0 \frac{\sin \omega_0 2\pi k}{\pi} \omega_0 2\pi k \end{array} \right| \]
Impulse Response

$h(kT)$ is of the form
Truncation

• To realise the frequency response we are required to use all of the infinite samples in the impulse response. But we are building a filter with a finite time response! We can do this by simply truncating the above series; we can set the coefficients past the fourth to zero by multiplying by a function which is 1.0 out to the fourth sample, and is zero elsewhere.

• Another word for truncation is windowing.
Impulse Response

Multiply

Response
**Frequency Response**

Note: In this case we are treating time as discrete and performing a circular convolution.
Comments

• The overshoot and ripple in the frequency domain is the price we pay for truncating the impulse response.
• It is the result of selecting the Fourier coefficients to minimise the mean squared error between the function and its Fourier series approximation.
• We, of course, recognise it as Gibb’s phenomenon.
• The poor behaviour in the neighbourhood of the discontinuity is the result of convolving with a “bumpy” function.
• Gibb's phenomena is not due to the fact that only a finite number of terms have been employed in the reconstruction!

• The magnitude of the peak ripple is invariant to the number of terms in the series.

• Adding more terms simply moves the ripple towards the discontinuity; it does not change its structure.
Gibb's Effect

The effect of adding more terms (lengthening the filter)

overshoot
• Note that ripple in the neighbourhood of 10% overshoot in the passband represents only 0.7 dB, but that 10% ripple in the stop band represents 20 dB.
• We would probably find this filter inadequate to meet filtering requirements in the stop band.
• How do we improve performance?
• We have commented that the poor ripple characteristics are due to the bumpy function used in the convolution.
• To reduce the ripple, we need a truncation function whose Fourier Transform is not as bumpy (i.e., smoother)
• The bumpy behaviour of the transform is related to the discontinuity of the truncation function.
• The study of smooth truncation functions classically referred to as windows is a very important, but not well-understood segment of DSP.
• Misconceptions abound!
• What follows is an assortment of common window functions followed by a series of important observations.
Normalised Rectangular Window

Spectrum of Rectangular Window

Narrowest main lobe

Sidelobe 13 dB down

winplot
Normalised Hamming Window

Spectrum of Hamming Window

Sidelobe
-41.6 dB down
Normalised Blackman Window

Spectrum of Blackman Window

Sidelobe -58 dB down
Normalised 3-Term Blackman-harris Window

Spectrum of 3-Term Blackman-harris Window

Sidelobe -66 dB down
Normalised Kaiser Window (Beta=5.65)

Spectrum of Kaiser Window (Beta=5.65)

Sidelobe -42 dB down
Normalised 4-Term Blackman harris Window

Spectrum of 4-Term Blackman harris Window

Sidelobe -92 dB down
Windows

- Windows with \textit{“smoother”} behaviour in the time-domain are \textit{“smoother”} in the frequency domain.

\begin{itemize}
  \item boxcar
  \item Bartlett
  \item Hann
  \item bkharris
\end{itemize}
Windows

- Windows that are “smoother” in the time-domain tend to have narrower time duration (e.g., smaller rms time-widths).

![Diagram of windows](image)

- boxcar
- Bartlett
- Hann
- bkharris
Windows

- Windows that are “smoother” in the time domain tend to have wider bandwidths (e.g., greater rms bandwidths)

[Diagram showing boxcar and bkharris windows with boxcar labeled as much wider than bkharris]
Windows

- When convolving with a wider function, the resultant function will have wider transitions (smaller slope).
Smoother Windows

• One mathematical interpretation of "smoother" is continuous derivatives.

[Diagram showing discontinuities, differentiation, integration, and Fourier Transform (FT) relating to sidelobe fall-off.

- Sidelobes fall off as $1/f$
- Divide by $j2\pi f$
- Sidelobes don’t fall off
Smoother Windows

- So a delta functions in the $n$th derivative implies that sidelobes fall off as $1/f^n$.

  - boxcar

  - Bartlett

  - windows get narrower

  - sidelobes fall off as $\frac{1}{f}$

  - $\frac{1}{f^2}$

  - $\frac{1}{f^3}$

Need continuous derivatives
**Sidelobes**

- Don’t want constant sidelobes.
- Why?
- When we downsample all the noise adds up.
- Always use windows which have some sidelobe fall-off. (About 3dB per octave is sufficient)
Transition Width

- The boxcar window gives the fastest rolloff in the transition band but has large sidelobes in the stop band.
- We trade rolloff for sidelobe suppression by using more sophisticated window functions.
Another View

- If we use a boxcar function, the rolloff in the transition zone is determined by the final sinc function and this is the fastest possible rolloff that can be achieved.
Optimal Windows

- **Dolph-Chebyshev**
  - Minimum main lobe width for lowest sidelobe level

- **Kaiser-Bessel**
  - Window is a good approximation to finite prolate spheroidal functions; minimum time bandwidth product
  - Family of windows with different sidelobe levels

- **Blackman-Harris**
  - Minimum sidelobe for a fixed number of spectral lines