1. Multiply the following polynomials in $z$ by using the fft algorithm (MATLAB).

\[
\begin{align*}
1 + 2z^{-1} + 4z^{-2} + 5z^{-3} + 11z^{-4} + 25z^{-5} \\
1 - 4z^{-1} - 5z^{-2} + 15z^{-3} - 13z^{-4} + 19z^{-5}
\end{align*}
\]

(10 marks)

2. Multiply the following base 10 numbers together and then show how the same result can be obtained via the DFT. Explain your method.

537283 multiplied by 423715

(10 marks)

3. Consider the polynomial

\[
F(z) = 1 + 2z^{-1} + 3z^{-2} + 5z^{-3} + 12z^{-4} + 25z^{-5}
\]

a) Write down and plot the positions of all the poles and zeros (MATLAB roots/zplane).

b) Evaluate $|F(z)|$ at 256 uniformly spaced points around the unit circle using the Discrete Fourier Transform and plot the result.

(10 marks)

4. Consider a 10 Hz sine wave sampled at 40 Hz. If the samples are plotted and displayed using linear interpolation, the reconstructed waveform will not resemble the original continuous-time sine wave. Show how sinc interpolation can be used to reconstruct the original continuous-time signal. (Hint: In this question you are being asked to resample the sine wave at a higher sampling rate)

(10 marks)

5. An N point DFT can be considered to be a set of N filters; each filter tuned to detect one particular frequency. Consider the filter corresponding to the DC component with $N=10$.

a) Plot the frequency response of this filter.

b) How many times does the magnitude of this filter go to zero around the unit circle?

c) Plot the zeros of the filter (use roots and zplane in Matlab)

(10 marks)

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6. An FIR filter with symmetric or antisymmetric coefficients will have linear phase. Show that for such a filter the zeros will either be on the unit circle or in reciprocal conjugate pairs. Hint – For symmetric coefficients show that

\[ H(z) = z^{-M} \left( H(z^{-1}) \right) \]

and examine the zero locations. (10 marks)

7. Design a LP FIR filter to meet the following specifications using the window method. Use a Blackman window.
   - Fs = 10 kHz
   - Fc = 1.5 kHz (3 dB down)
   - Attenuation = 60 dB at 2.5 kHz

Give all the relevant plots (impulse, frequency responses) and the performance of the final filter. Compare this filter to one designed using the optimal (remez) method. (20 marks)

8. Design a HP FIR filter to meet the following specifications using the window method. Use a Kaiser window.
   - Fs = 10 kHz
   - Fc = 3.5 kHz (1 db down)
   - Attenuation = 60 dB at 3 kHz

Give all the relevant plots (impulse, frequency responses) and the performance of the final filter. Compare this filter to one designed using the optimal (remez) method. (20 marks)